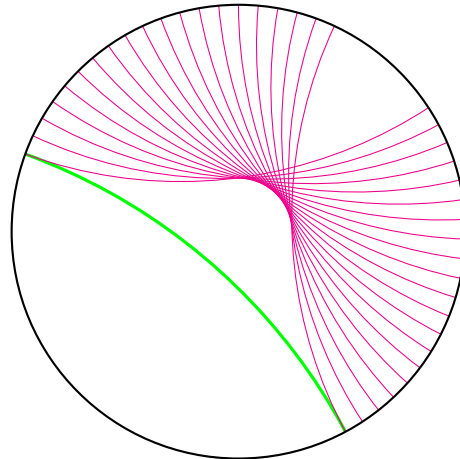




UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

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July 20, 2021

HEGL PROSEMINAR / SEMINAR VISUALIZATIONS IN HYPERBOLIC GEOMETRY



This is the second iteration of the HEGL Seminar, the seminar of the newly founded **Heidelberg Experimental Geometry Lab (HEGL)**. In the upcoming winter semester, we will focus on the visualization of fundamental objects and transformations of hyperbolic geometry in different models of the hyperbolic plane.

Participation in this seminar includes:

- Giving a (shorter) mathematical talk,
- Working on a mathematical project in a small group,
- Giving a presentation of the project.

Seminar Topic:

Hyperbolic geometry is the star of non-Euclidean geometries. It occurs in a perfect “world” of constant negative curvature. It is a very active field of mathematical research and it can be useful in other sciences, including data science and machine learning.

You will learn about and manipulate basic notions of hyperbolic geometry such as: geodesic lines and segments, hyperbolic triangles and ideal triangles, reflections and orientation-preserving isometries horocycles.

The goal of the seminar is to experiment with these concepts in the different models of the hyperbolic plane, and to understand the relations between the models (stereographic projections, Cayley transform. In order to do this, we will use software and programming: GeoGebra, Python, Blender, etc; and 3D printing.

Seminar location and time:

Meeting on Friday, July 23, 2021 at 13:00 on Zoom to discuss backgrounds.

The topics will be decided after the first meeting.

More information will be sent to the interested students by email (so enroll early).

Preliminary schedule of the semester:

- Week 42: Introduction
- Weeks 43–45: Math talks (3 talks per session)
- Weeks 46–2: Work on projects
- Week 49: Short updates on projects
- Weeks 4–6: Final presentation of projects

The participants will be divided in groups of three (approximately). Each group will do one Math talk session, consisting of three individual talks, and one Project presentation.

Intended audience and prerequisites:

The intended audience is mainly Bachelor students, possibly Master students. Some background in programming is helpful but not required.

The language of the seminar will be English.

Enroll in the seminar:

If you are interested in this seminar, enroll in MÜSLI **before July 22** or contact Brice (bloustau@mathi.uni-heidelberg.de) or Anja (arandecker@mathi.uni-heidelberg.de).

List of topics:

Topic A: Introduction to hyperbolic geometry

Talk 1 Introduction to the main concepts of (hyperbolic) geometry: points and lines, line element and distances, angles, Riemannian and metric isometries.

Talk 2 Axiomatic approach to Euclidean and hyperbolic geometry. Discussion of Euclid's fifth postulate. Brief introduction to Poincaré disk.

Talk 3 Introduction to the concept of curvature.

Project 3D printing of hyperbolic objects (triangles, horocycles, or others to be determined). Realization in Blender + Python.

Topic B: The Poincaré models

Talk 4 Poincaré half-plane, part I: the basics (line element, distance, geodesics)

Talk 5 Poincaré half-plane, part II: more geometry (isometries, triangles, horocycles)

Talk 6 The Poincaré disk and its relation to the half-plane (Cayley transform).

Project Visualization in Python of objects and transformations in both Poincaré models, and going from one to the other.

Topic C: The hyperboloid and the Klein models

Talk 7 Minkowski space and the hyperboloid

Talk 8 The hyperboloid model (metric, distance, geodesics, isometries, curvature)

Talk 9 Stereographic projection to the Klein disk and the Poincaré disk.

Project Visualization in Python of the relations between the Poincaré disk, the hyperboloid model, and the Klein model.