Sudents searched for projects
Visualizing subriemanniann billiards

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Setting

Consider a space M. The distance between two points in M is determined by the length of the shortest path connecting them. In subriemannian geometry the distance is made a bit more interesting by only allowing a certain set of directions ξ for the velocity of the connecting path. The allowed directions at a point x form a linear subspace of velocities $\xi_x \subseteq T_xM$. The subspace ξ_x varies with the point x (ξ wiggles when moving x), so that one can still connect any two points by curves tangent to ξ. In this setting, it is still possible to define 'straight' lines (geodesics), although this geometry and dynamics is vastly different than in the usual Euclidean setting.

For a wall $W \subseteq M$, we can define a billiard reflection law if the intersection $\xi_x \cap T_xW$ is transverse. It is also possible that the billiard reflection law is degenerate: This happens if the subspace ξ is tangent to the wall $\xi_x \subseteq T_xW$, in which case no reflection law can be defined. We call such catastrophic points critical.

The geometry of these spaces is rather strange, and the reflection law even more so. In the projects proposed here, we intend to recognize some of the features of the billiard dynamics by 'just' looking. In order to 'just' look, we intend to produce visualizations of the billiard flow for specific situations in three dimensions.

Here are some features we are interested in:

- Qualitative behavior of the flow near critical points,
- Statistical behavior of billiard trajectories,
- Effectiveness of traps.

This gives the opportunity for multiple projects at different levels of study. The tasks involve the understanding of these features, how to recognize them through visualization and the visualization itself.

Requirements

Basic proficiency in C++, numerics and analysis, as obtained in the first two years of mathematical studies.

Learning opportunities

On the practical side, this is a first step into the world of visualization of dynamical systems.

On the theoretical side, one can learn about Hamiltonian dynamics, control theory and contact geometry.

Contact

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