

PROJECT
WALLPAPER GROUPS
IN THE MATHEMATIKON

CAROLINE, JOHANNES, SEBASTIAN



CLASSIFICATION

P or C

P: oblique,
rectangular,
square,
hexagonal

C: centred
rectangular

numbers

2: 180° turn (half-turn)

3: 120°

4: 90° turn

6: 60° turn

m and g

m: mirror
(reflection line)

g: glide
reflection

mm: mirrors in
2 directions

gg: glide
reflections in 2
directions

Largest order of rotation?

None

2

3

4

6

Is there a reflection?

Is there a reflection?

Is there a reflection?

Is there a reflection?

Is there a reflection?

Is there a glide reflection?

Is there a glide reflection?

Are these reflections in two directions?

Is there a glide reflection?

Are all rotation centres on mirror lines?

Are all rotation centres on mirror lines?

Are there mirror lines intersecting at 45°?

cm

pm

pg

p1

p2mm

c2mm

p2mg

p2gg

p2

p3m1

p31m

p3

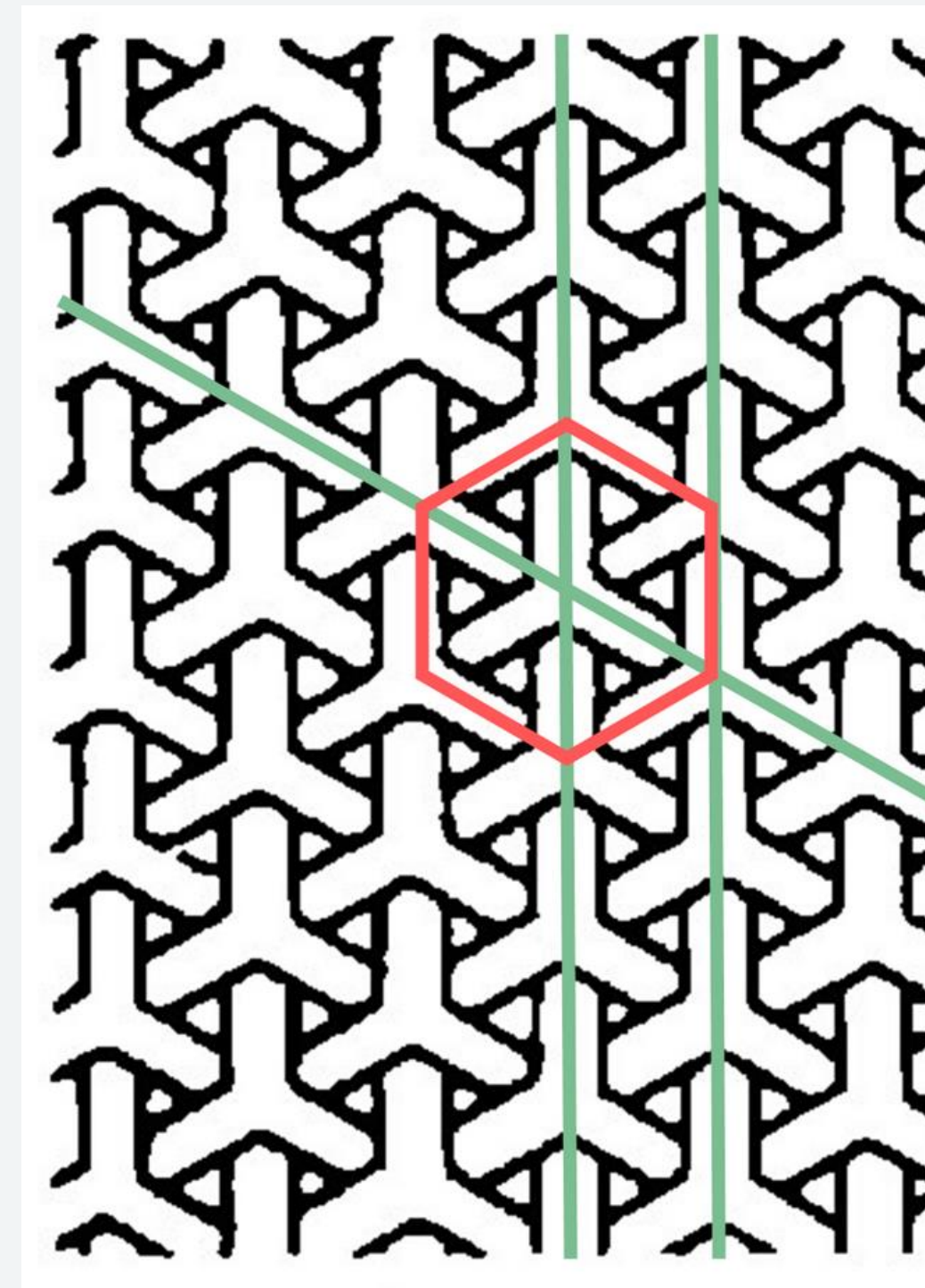
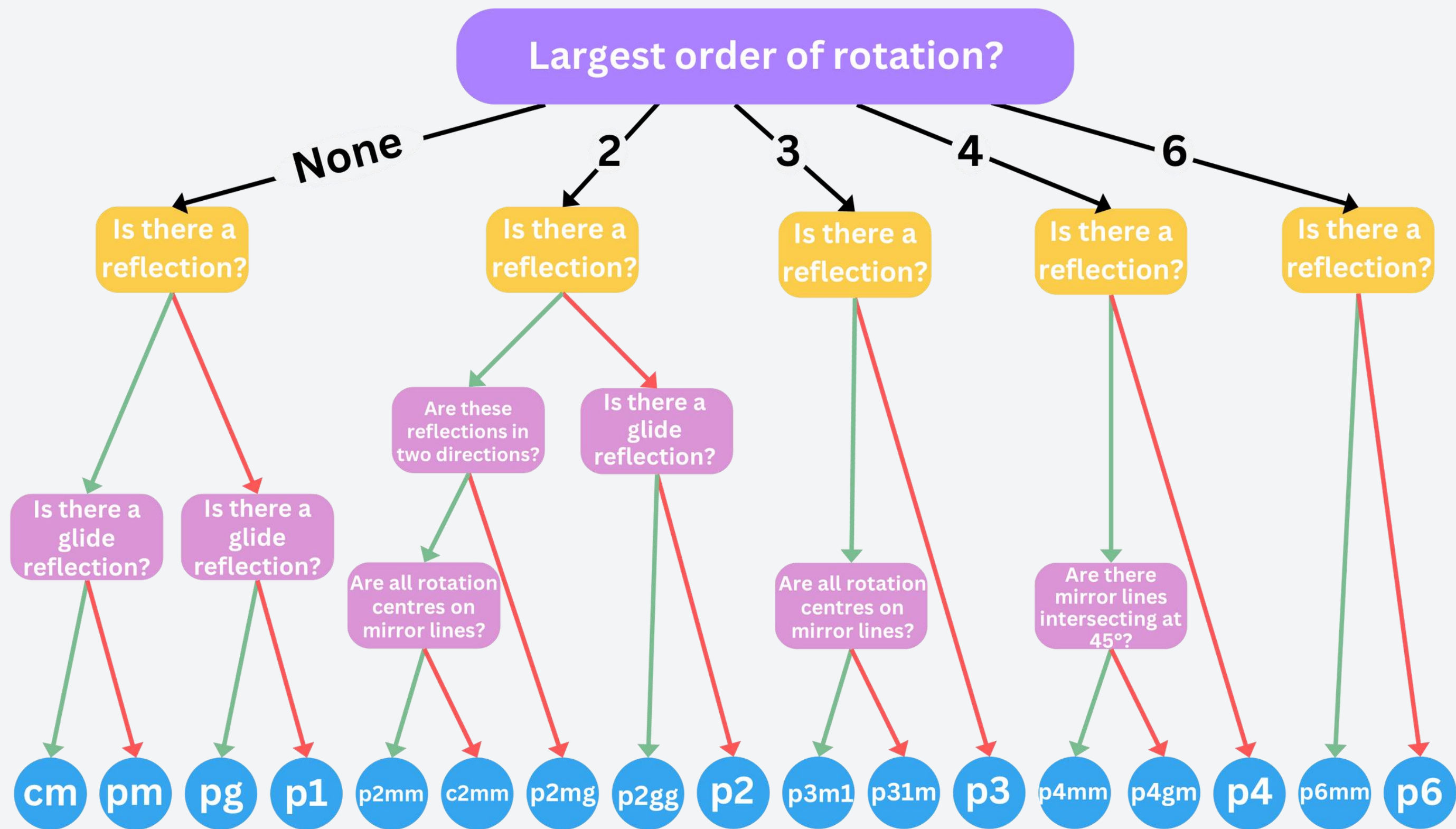
p4mm

p4gm

p4

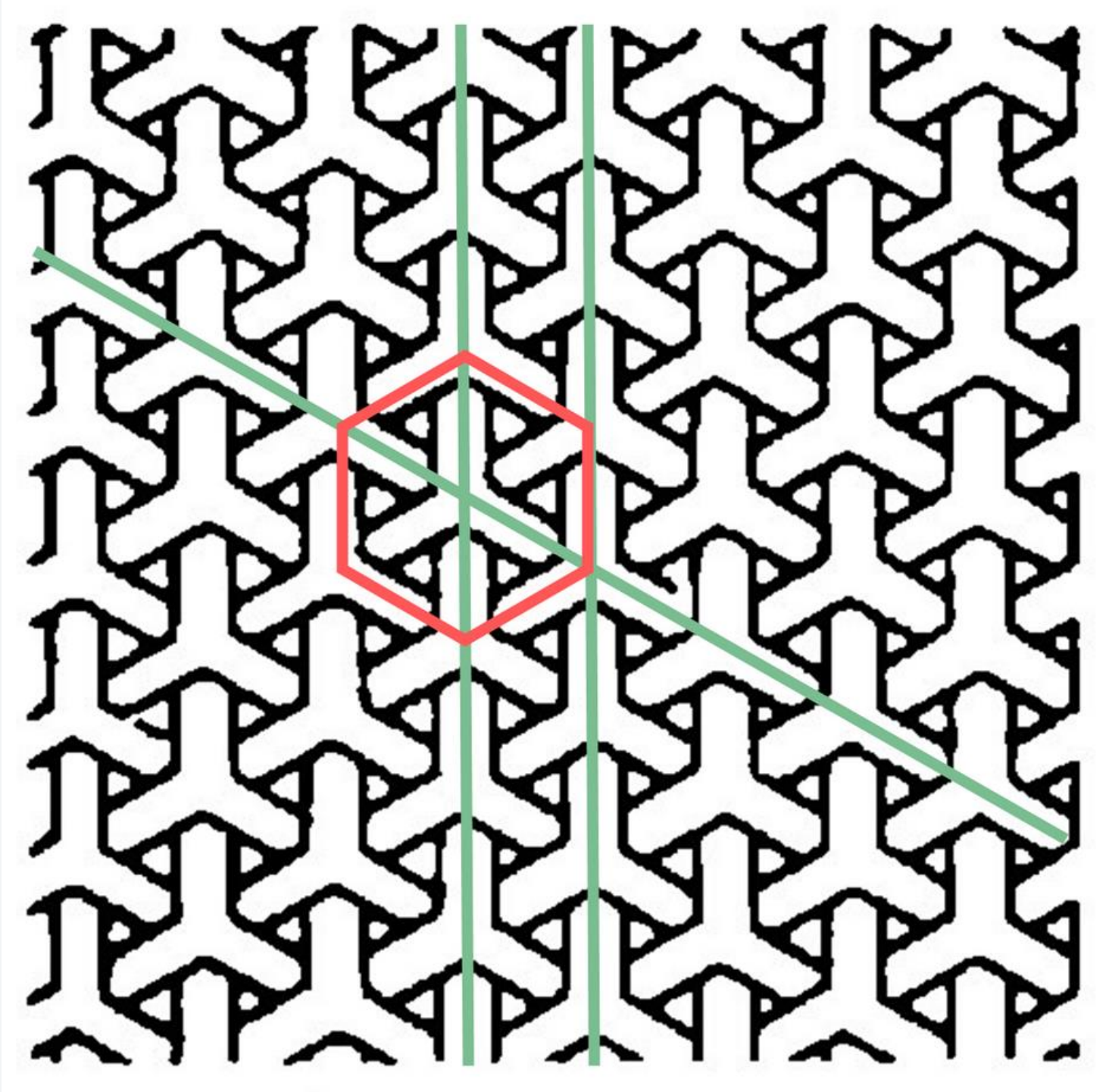
p6mm

p6



Largest order of rotation?

3



Is there a reflection?

Are all rotation centres on mirror lines?

- p3m1
- p31m
- p3

OBLIQUE



P1

P1

P2

OBLIQUE

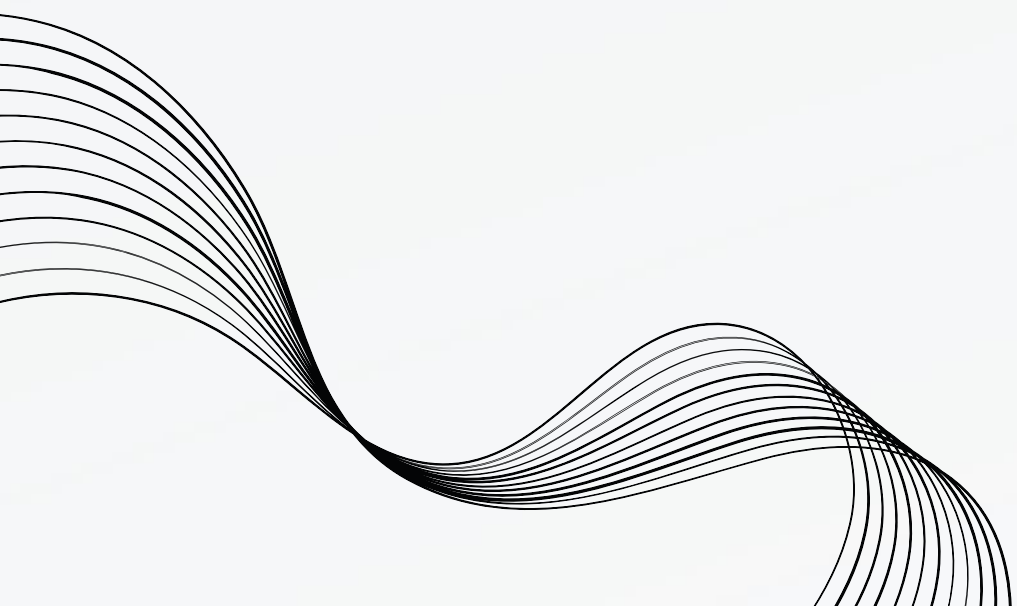
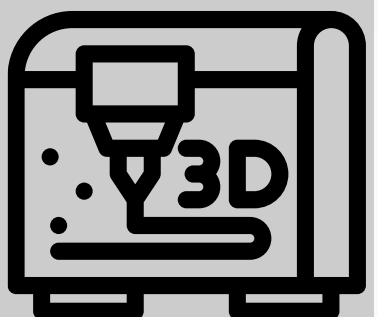
9+6 9+6 9
9x6 9x6
6 9+6 9+6
6 9x6 9
9+6 9+6 9



P1

P2

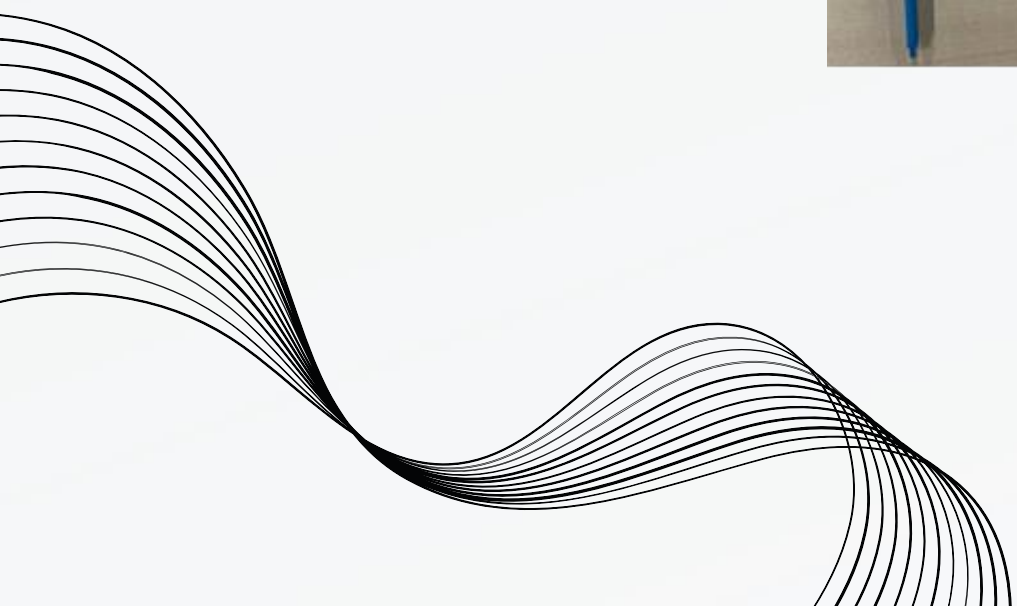
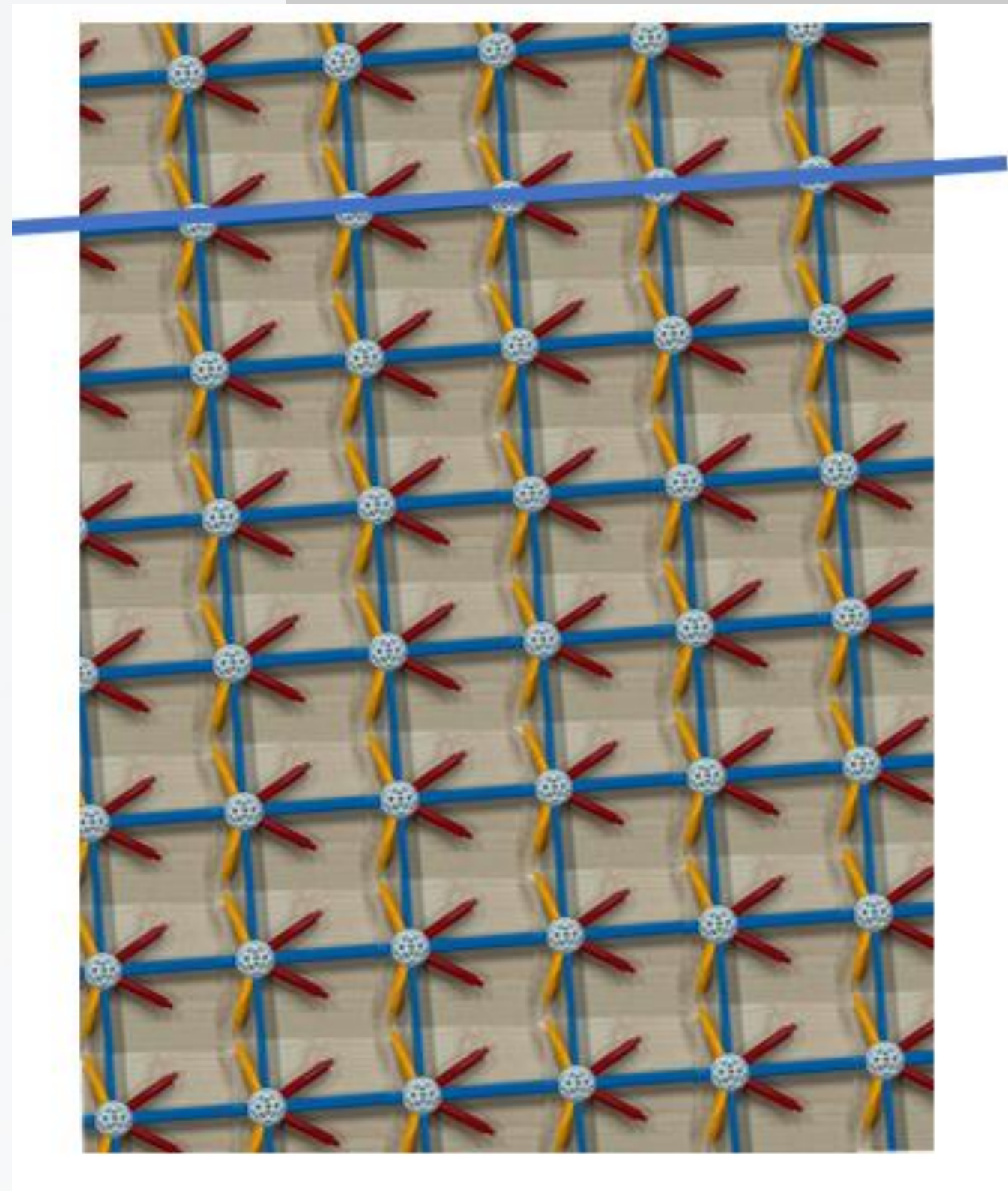
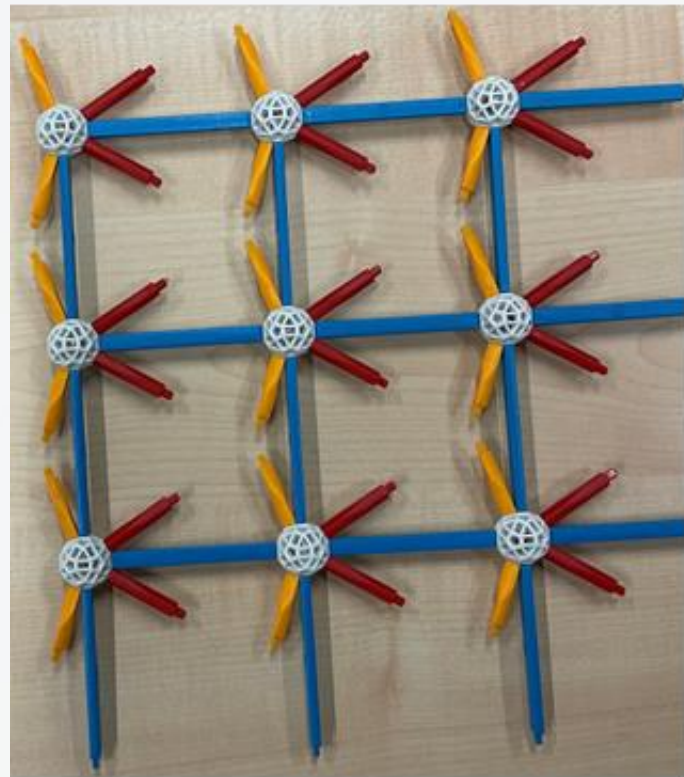
P2



RECTANGULAR

PM PG P2MM P2MG P2GG

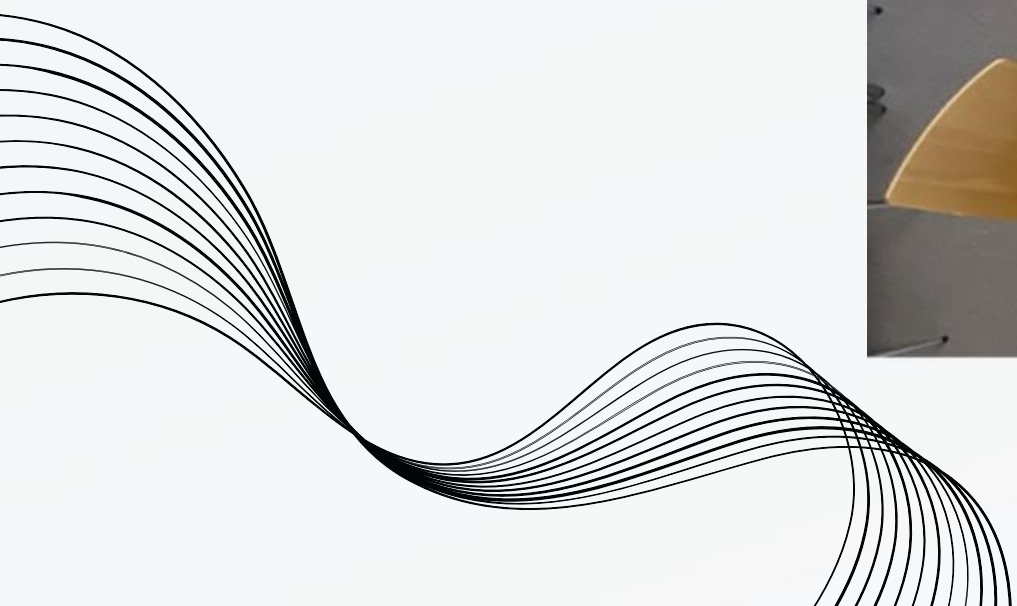
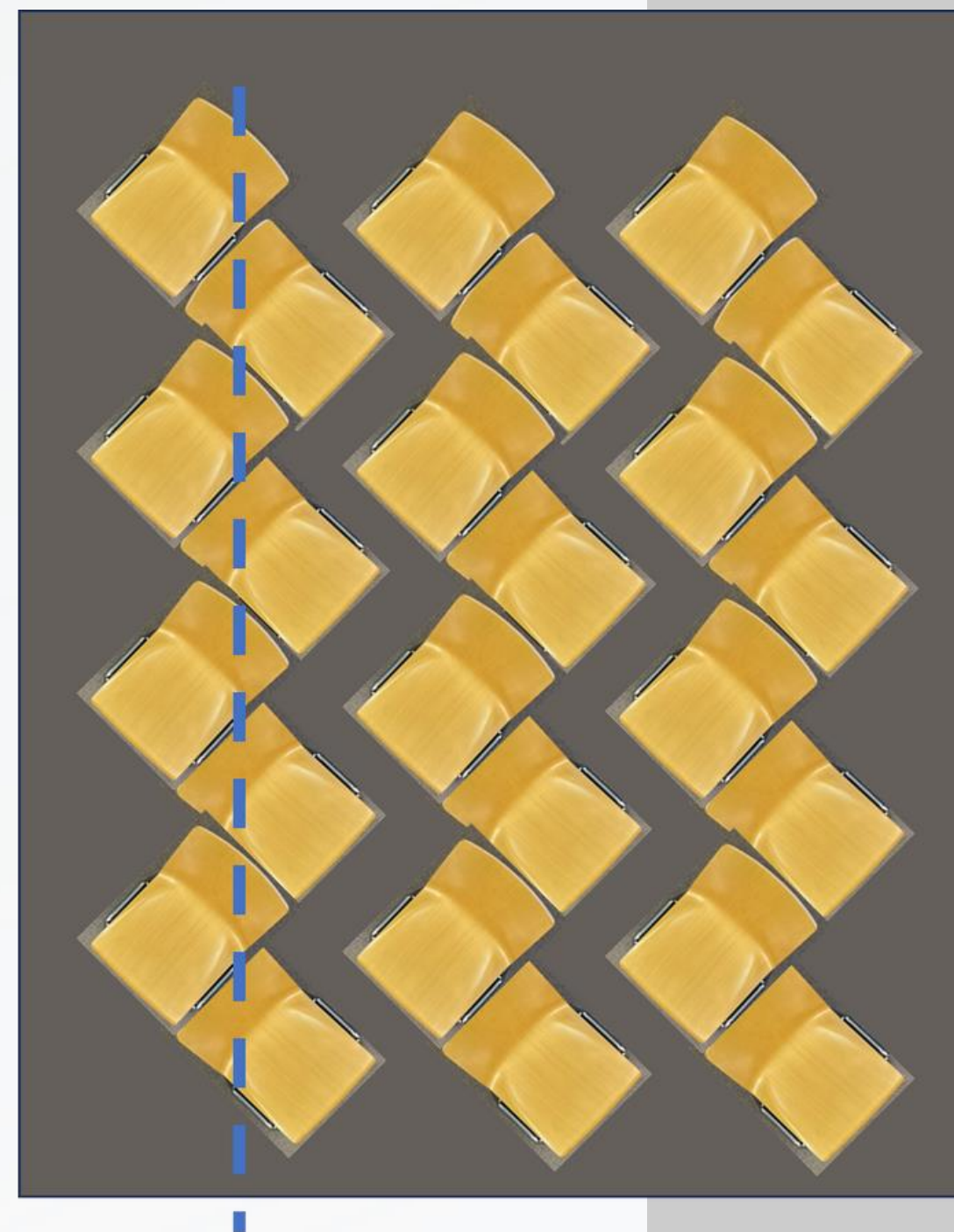
PM



RECTANGULAR

PM PG P2MM P2MG P2GG

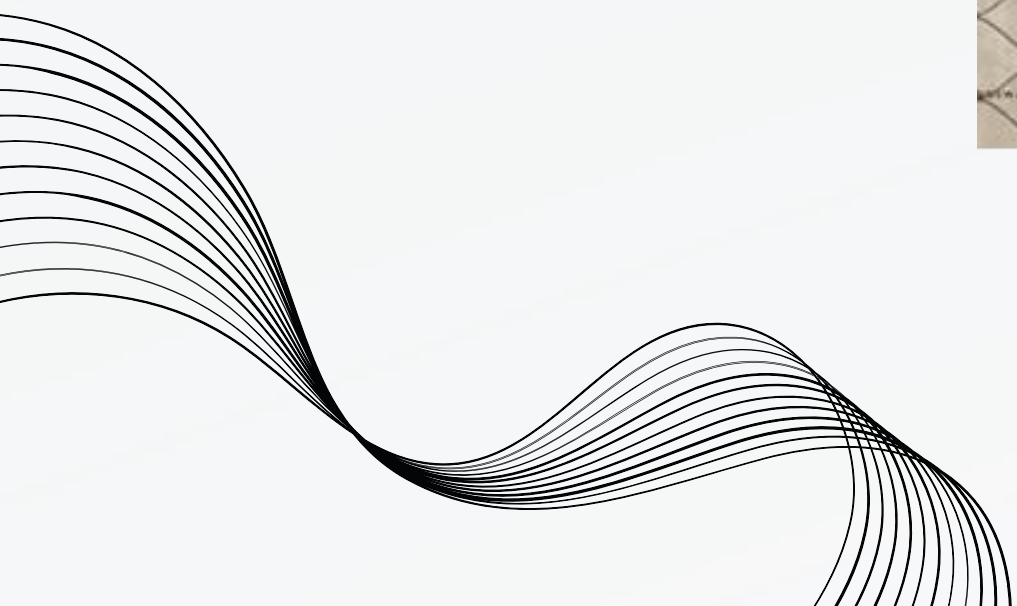
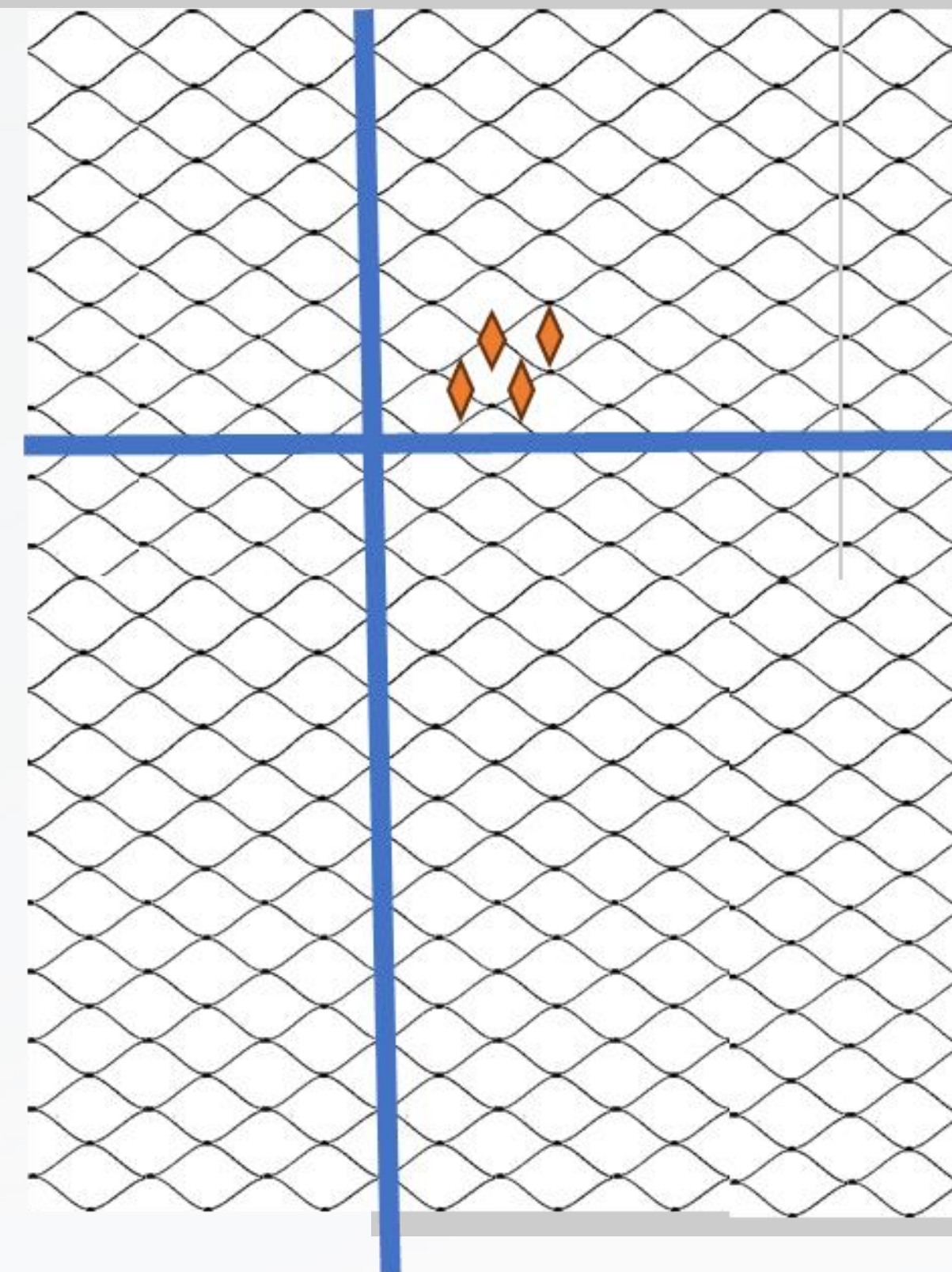
PG



RECTANGULAR

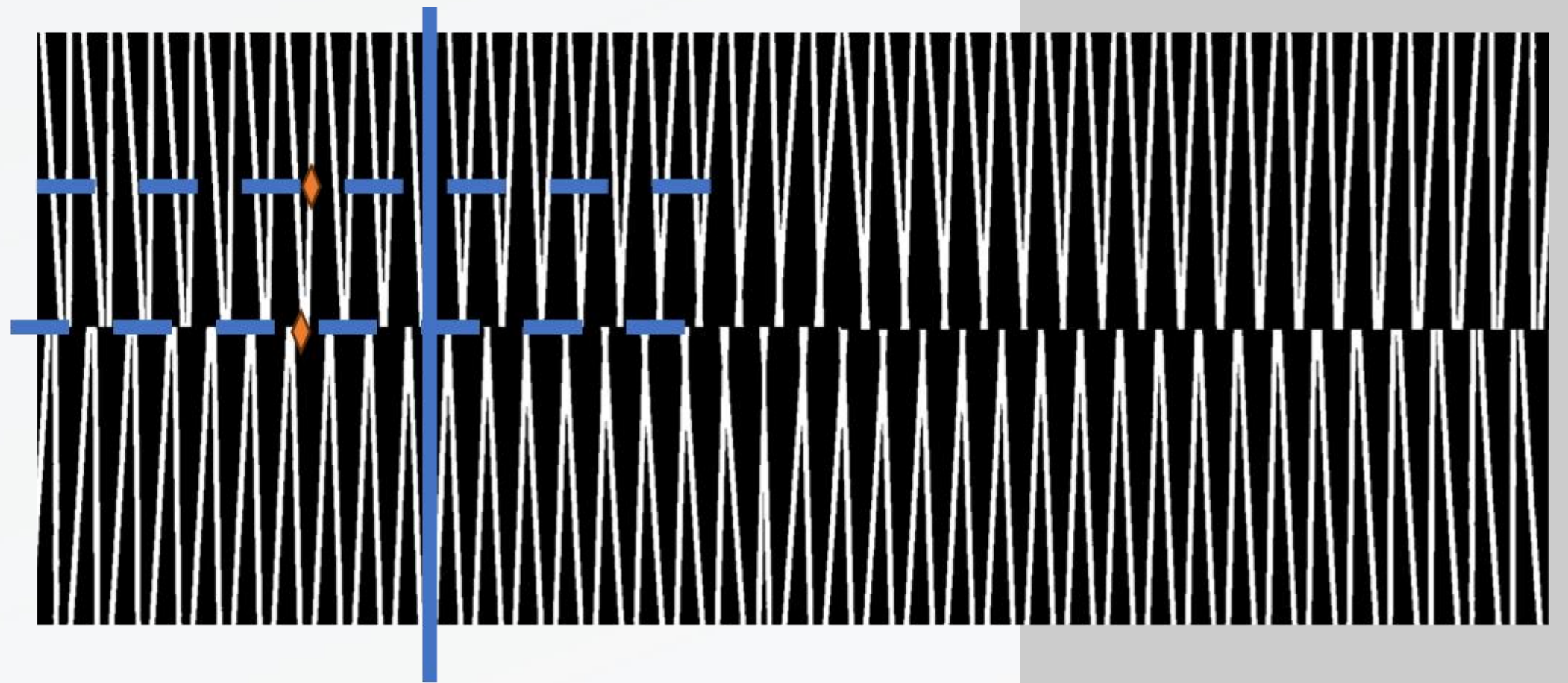
PM PG P2MM P2MG P2GG

P2MM

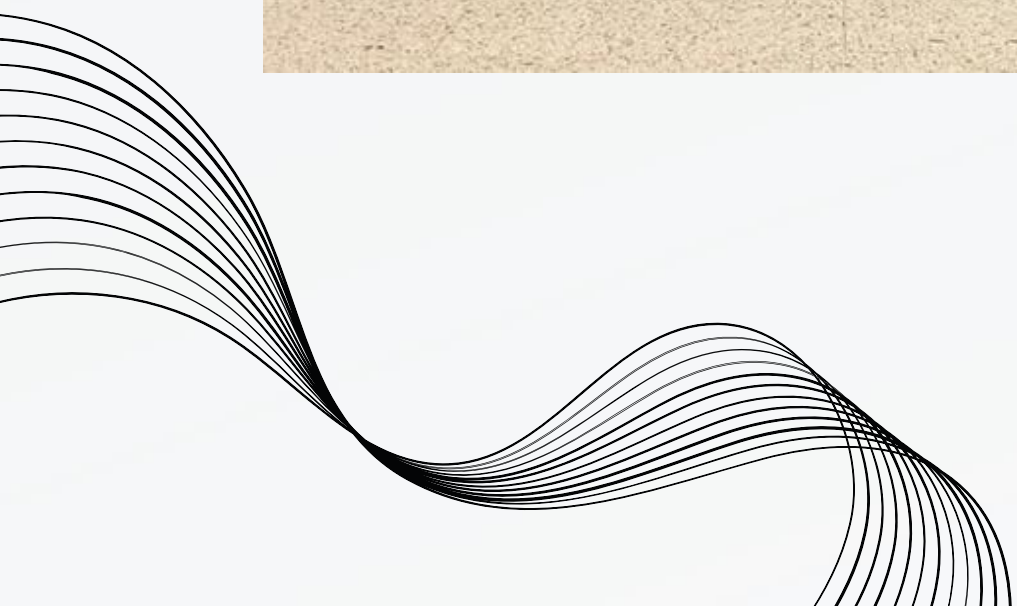


RECTANGULAR

PM PG P2MM P2MG P2GG

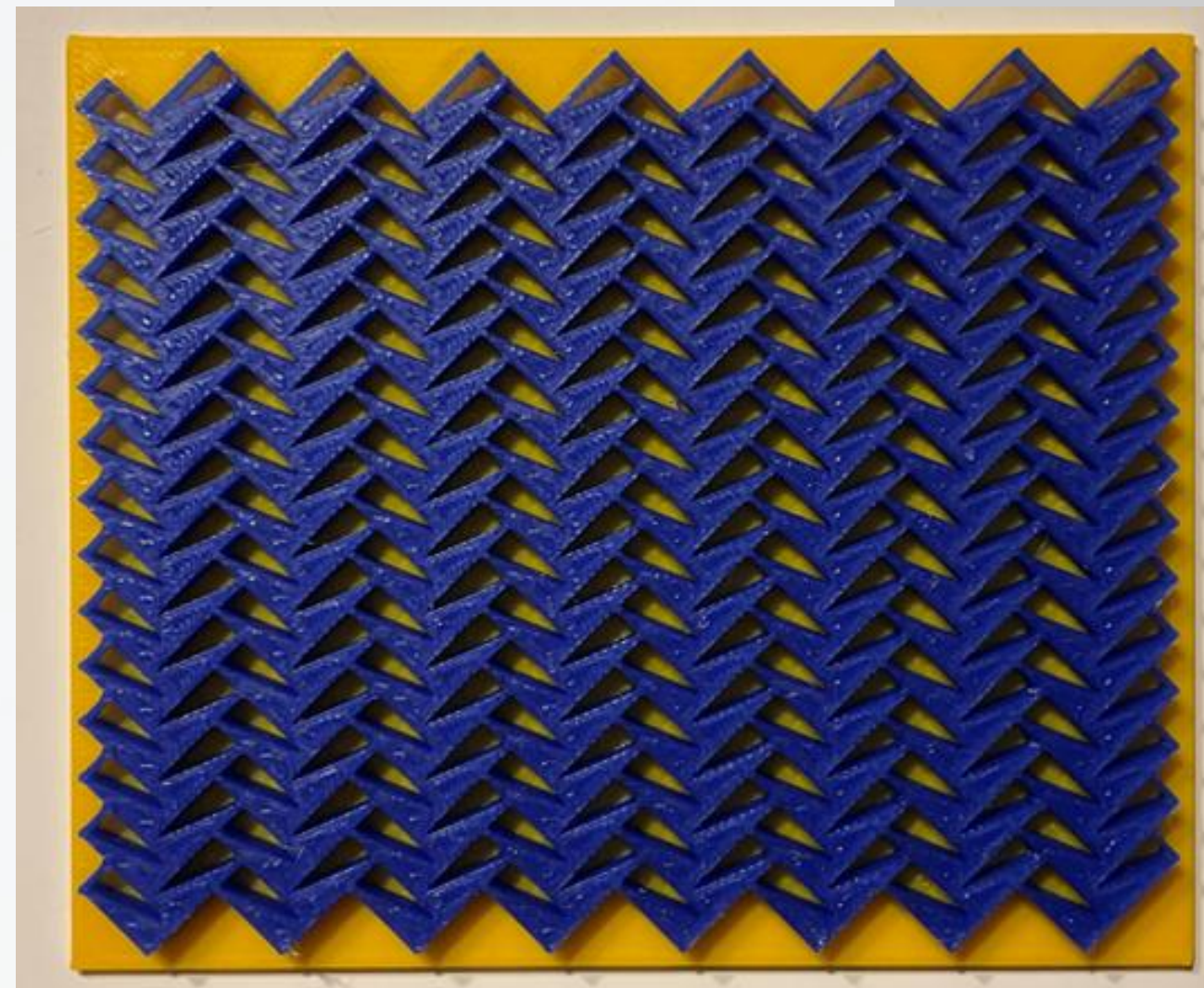
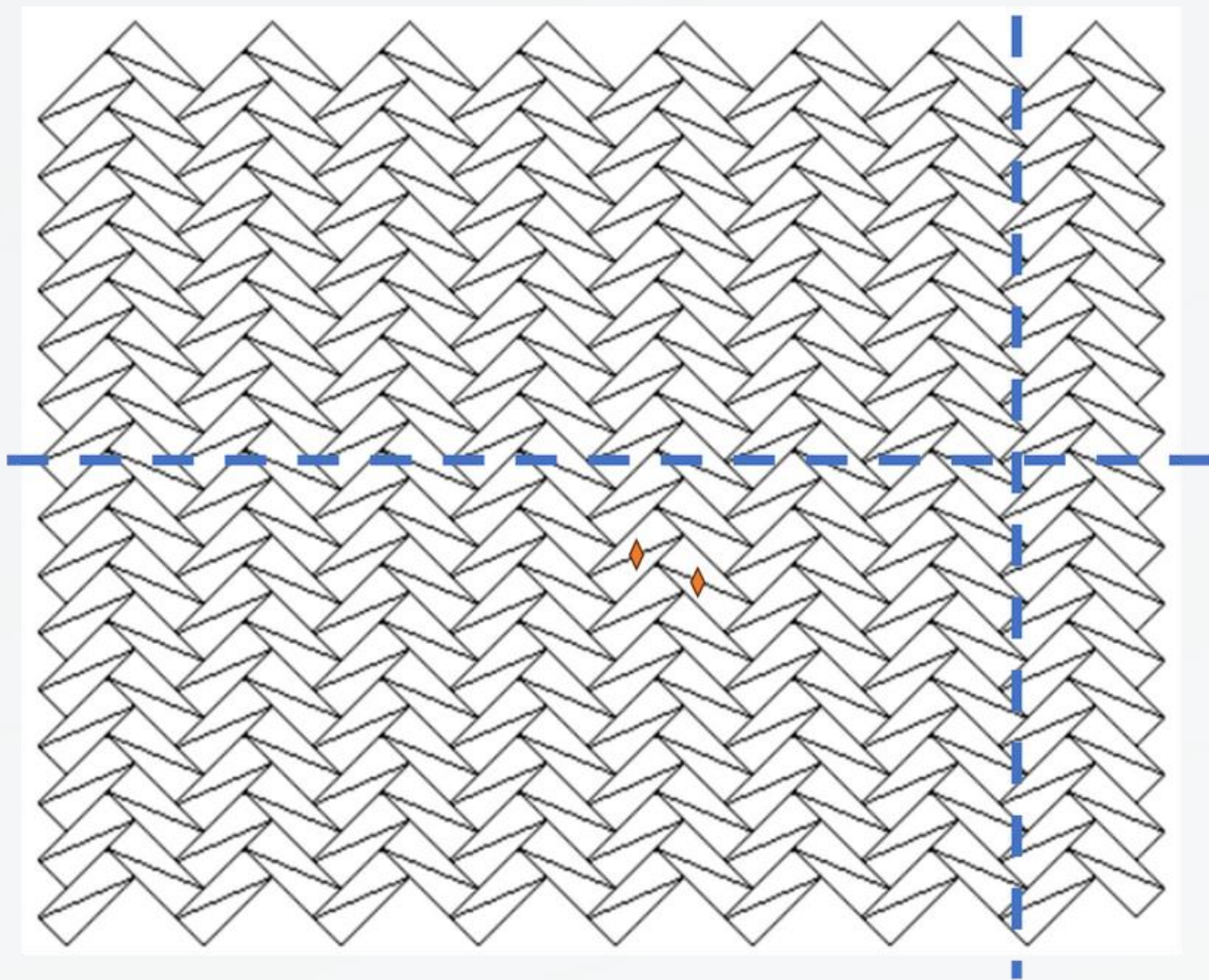


P2MG

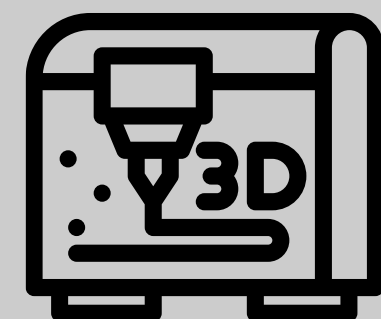


RECTANGULAR

PM PG P2MM P2MG P2GG



P2GG

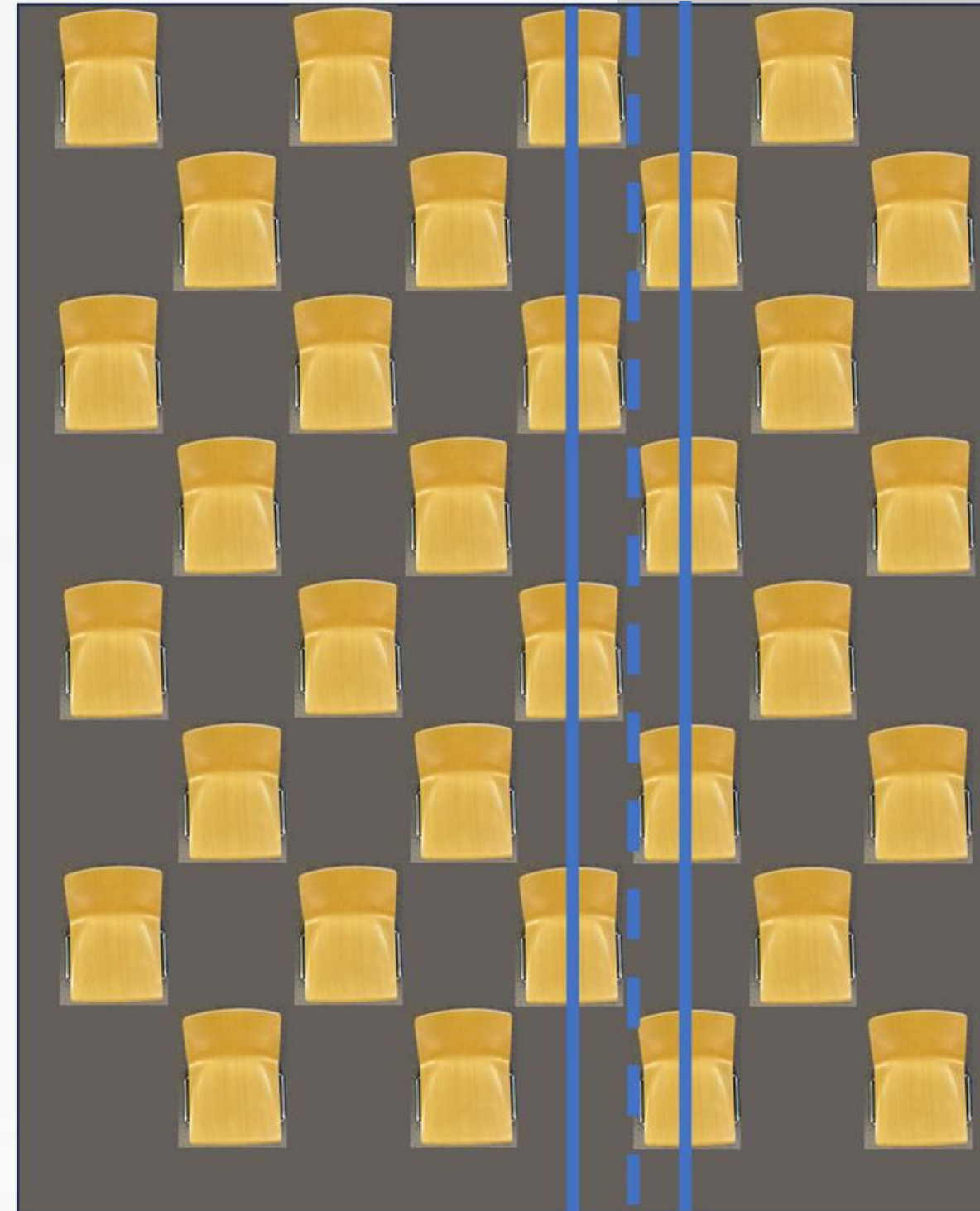


CENTRED RECTANGULAR

CM

C2MM

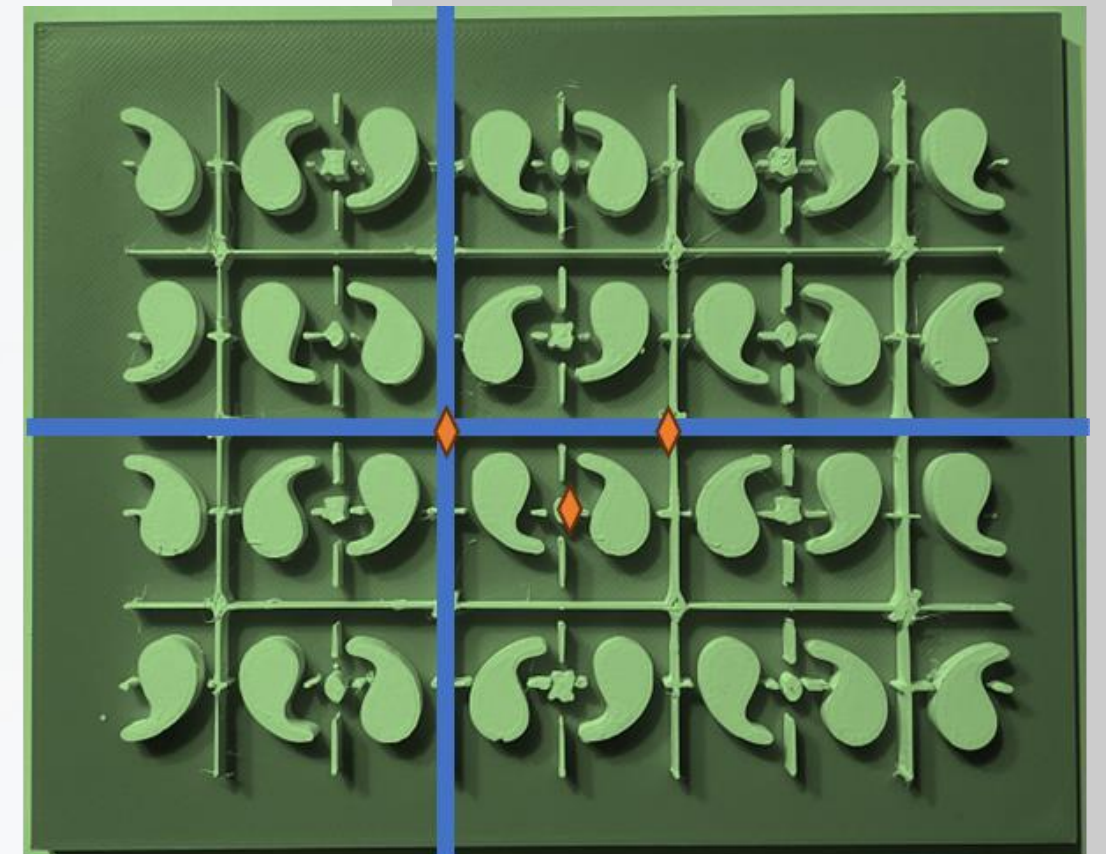
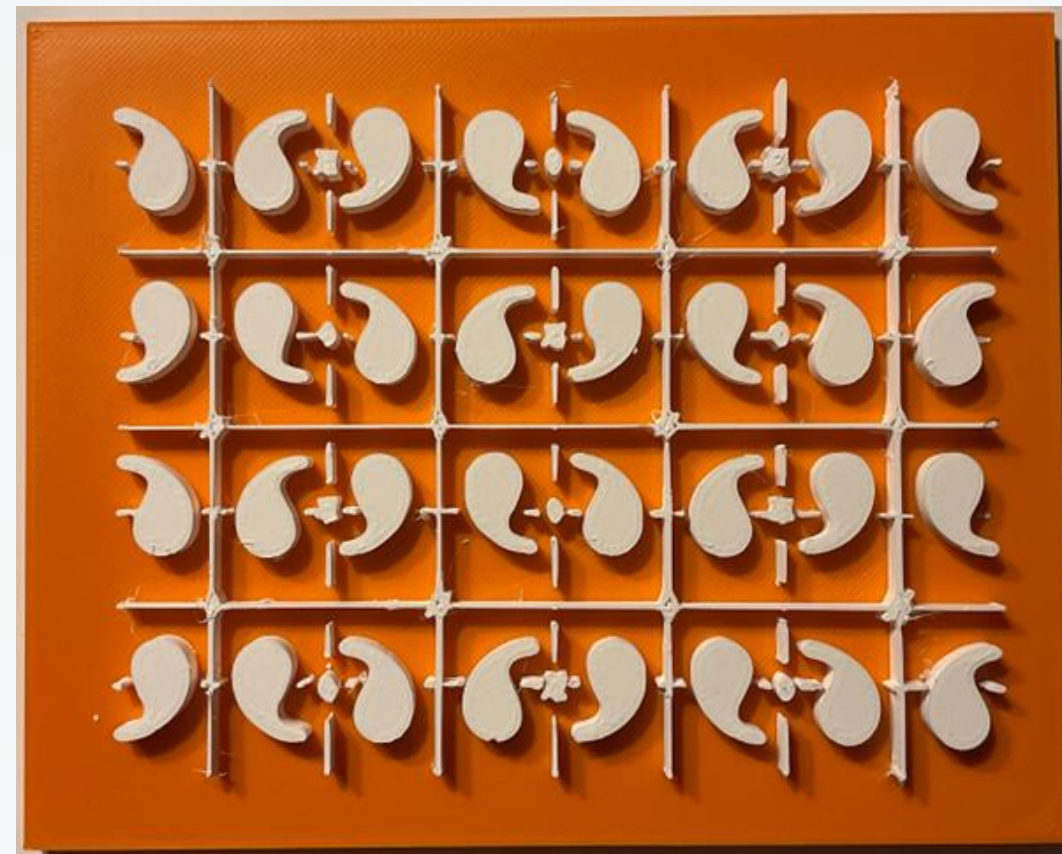
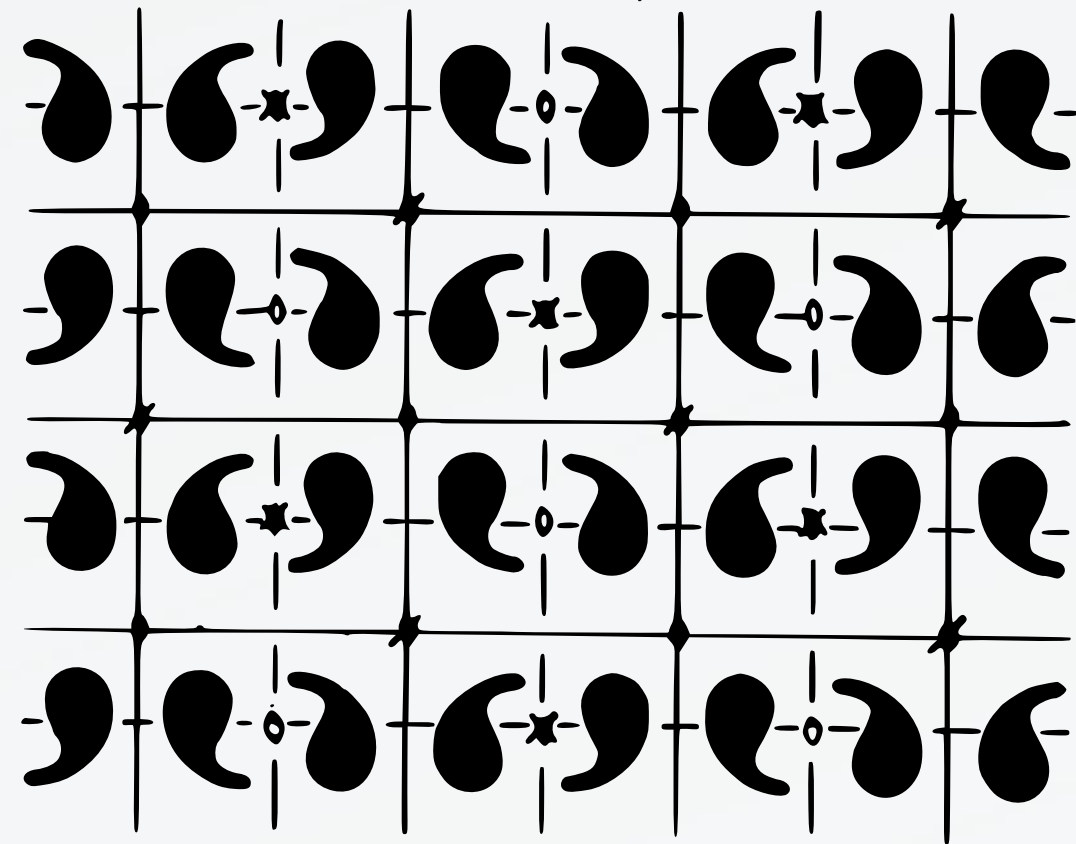
CM



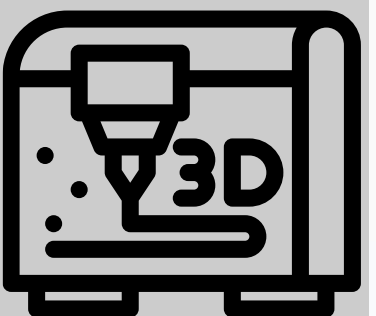
CENTRED RECTANGULAR

CM

C2MM



C2MM



SQUARE

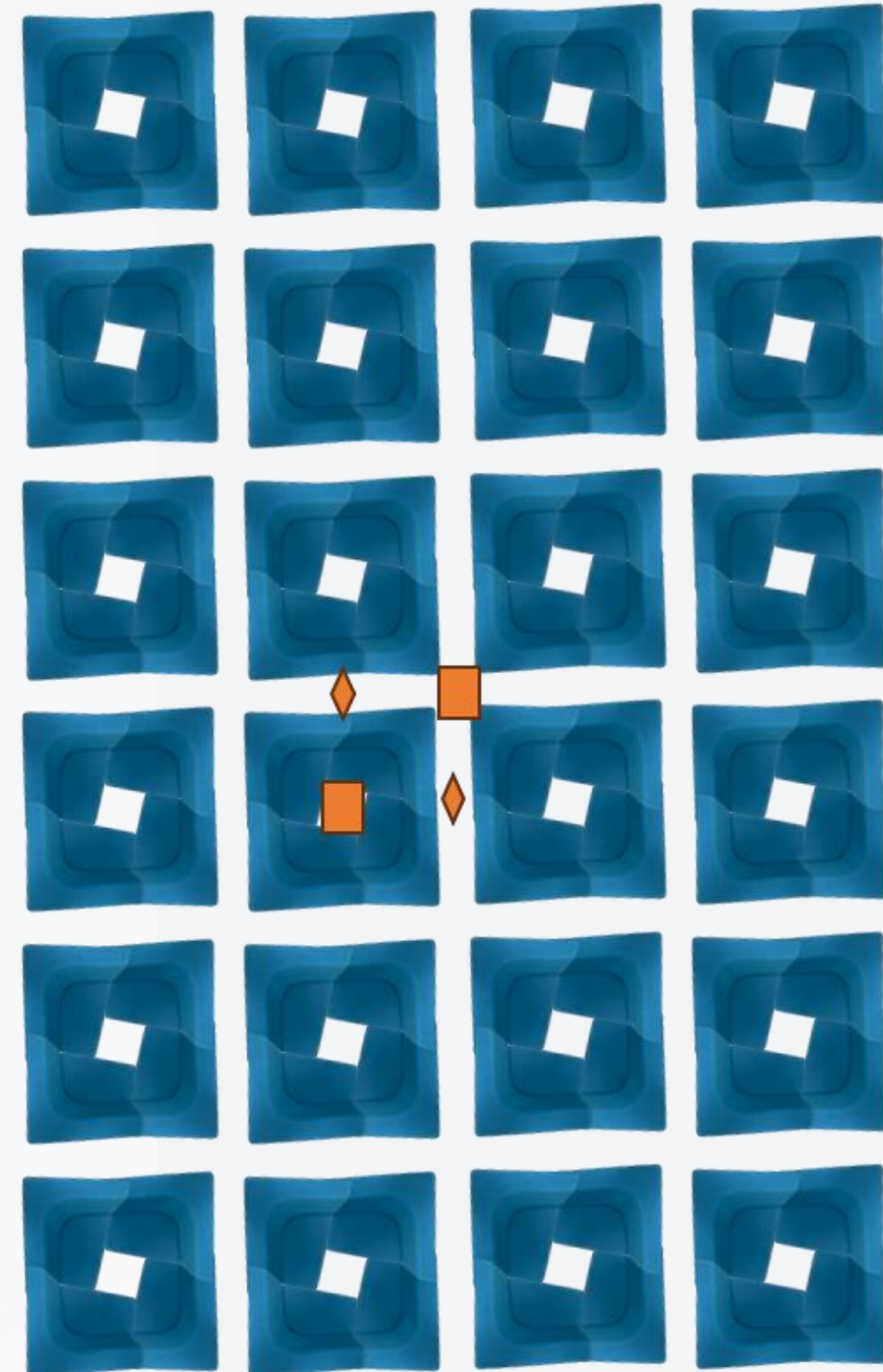
P4



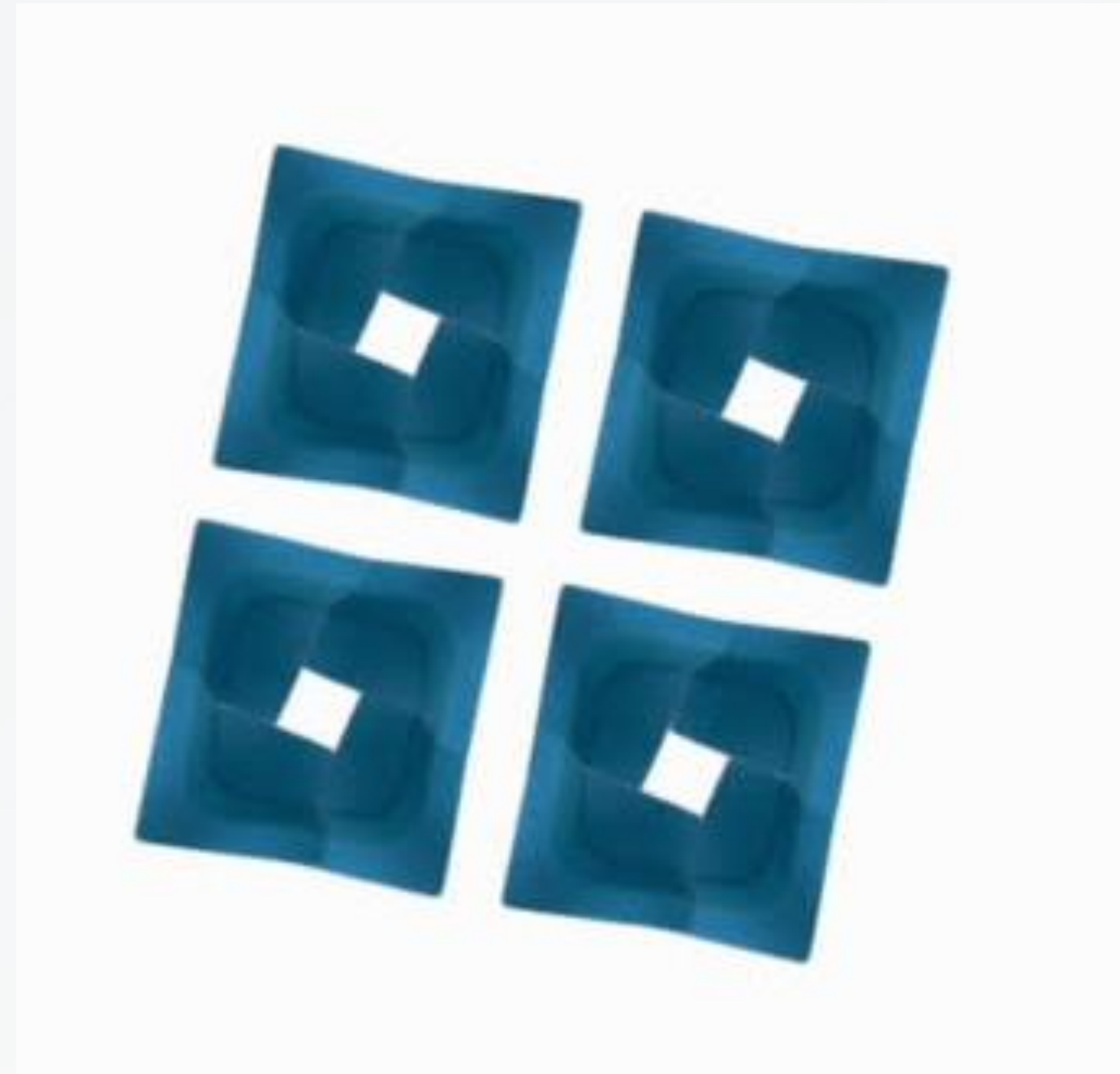
P4

P4MM

P4GM



ANIMATION P4



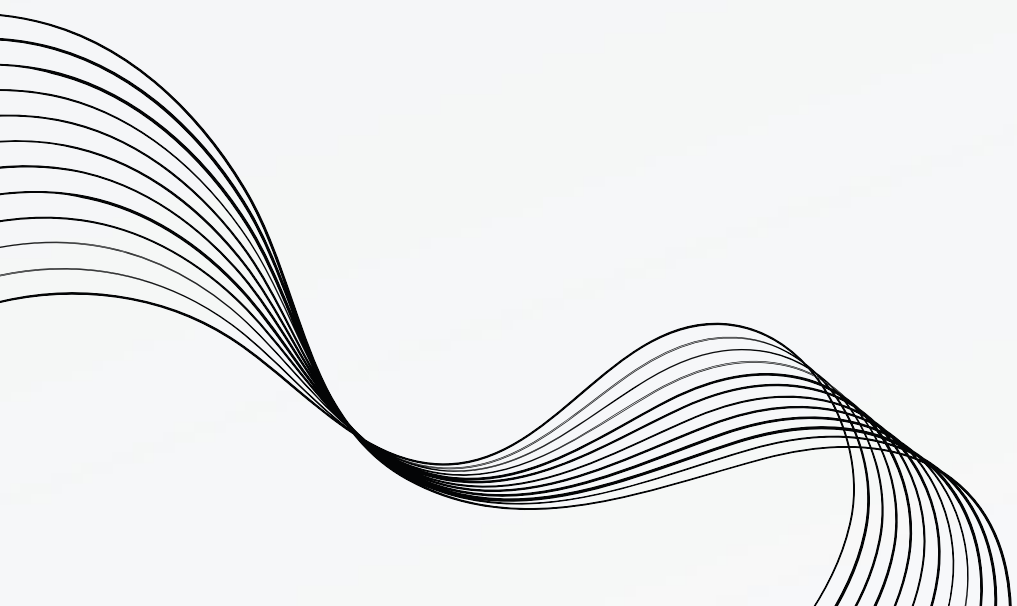
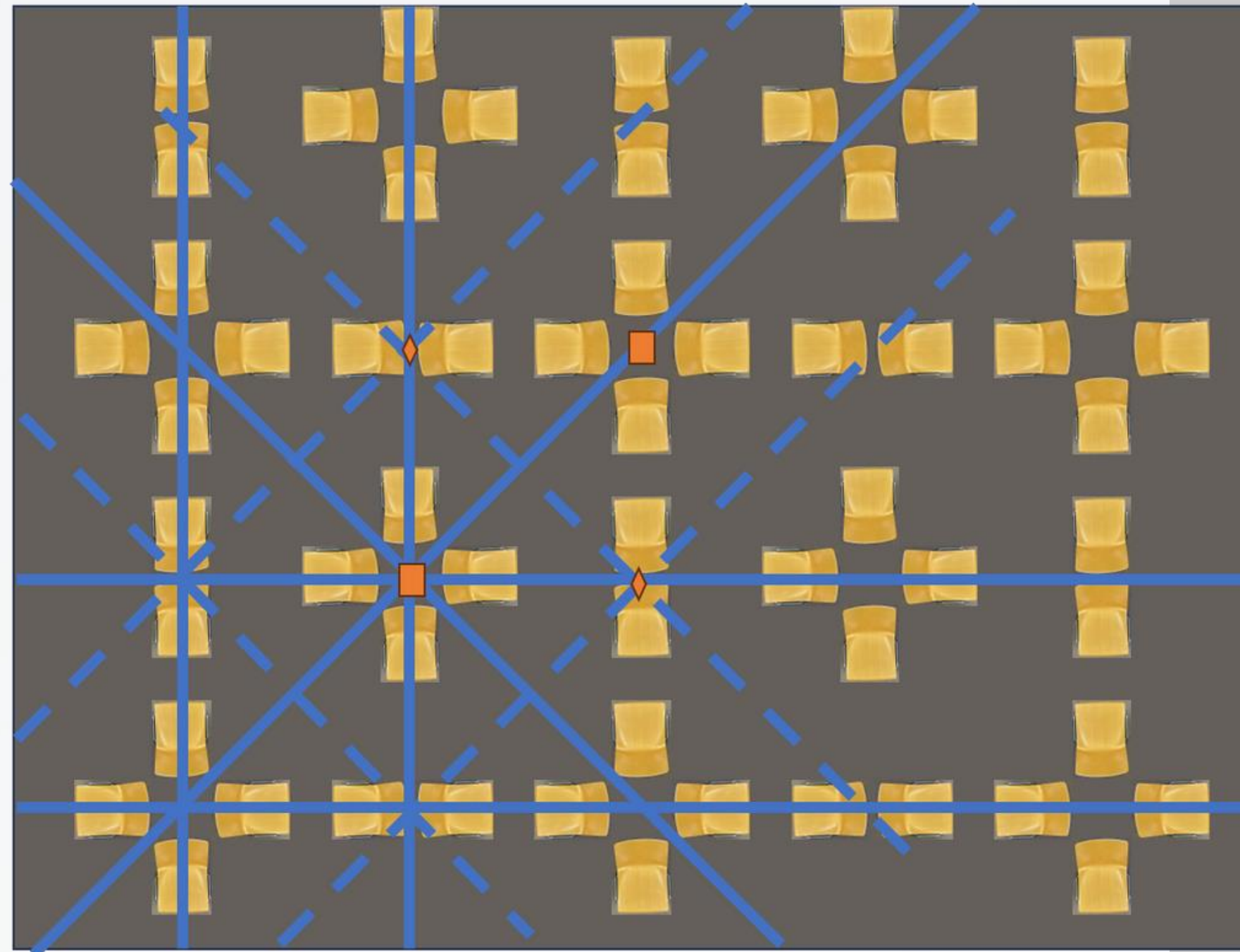
SQUARE

P4

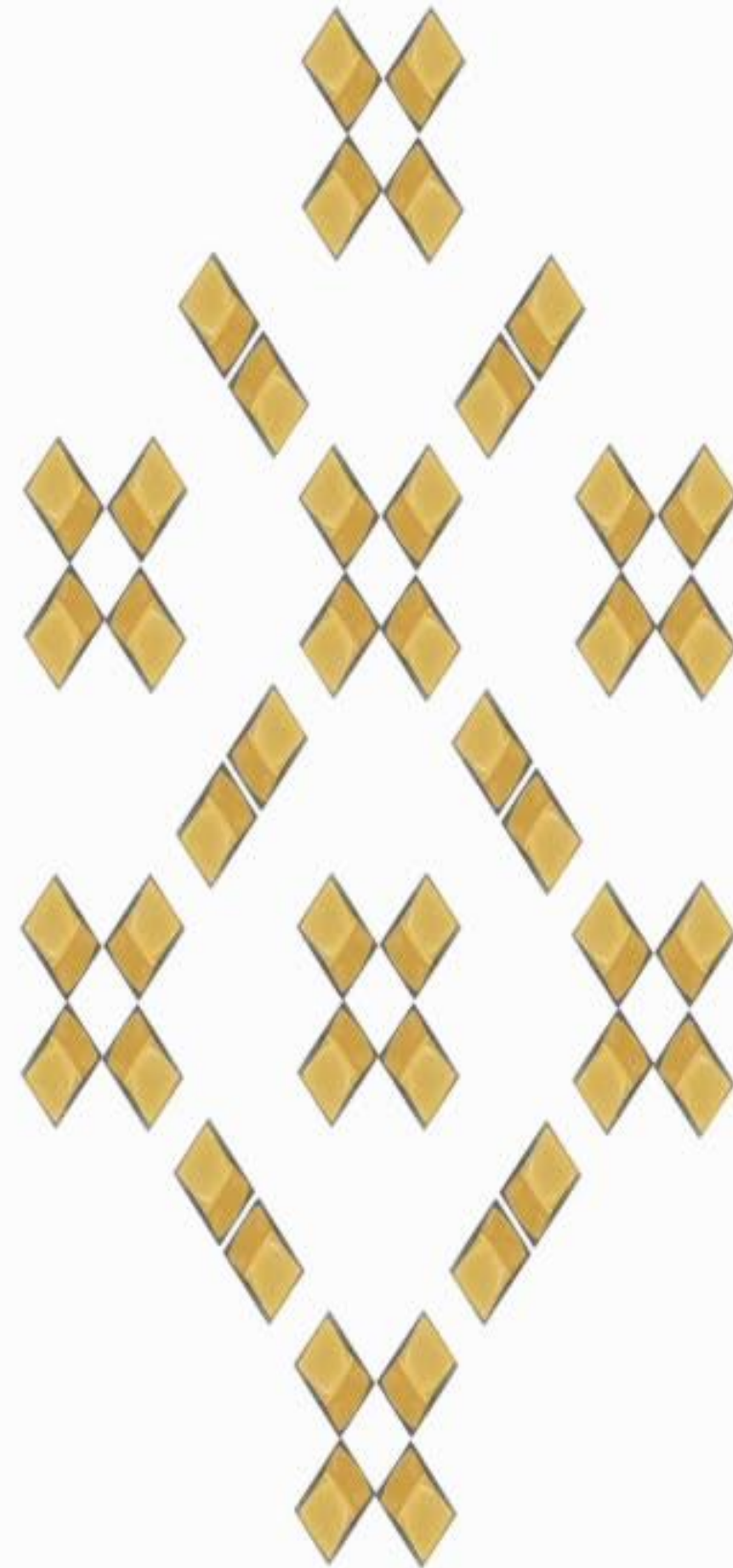
P4MM

P4GM

P4MM



ANIMATION P4MM

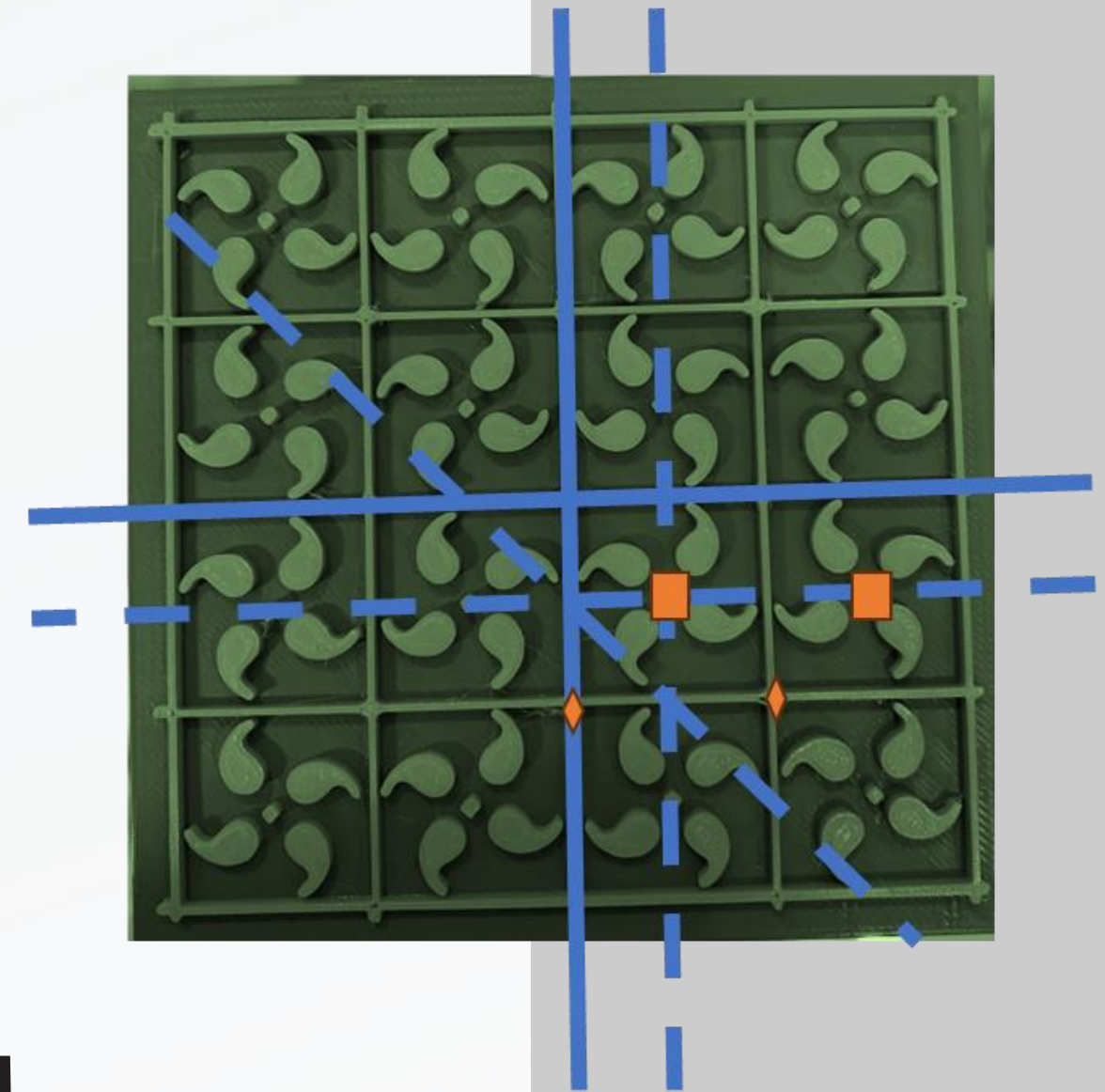
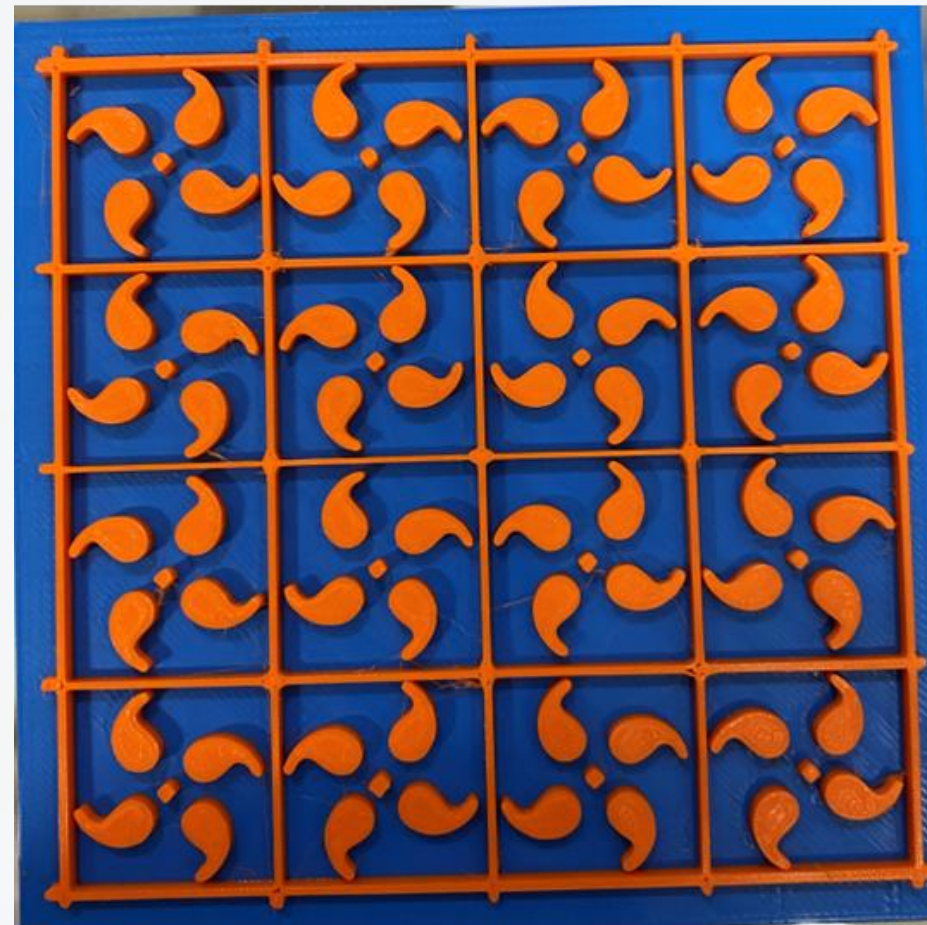
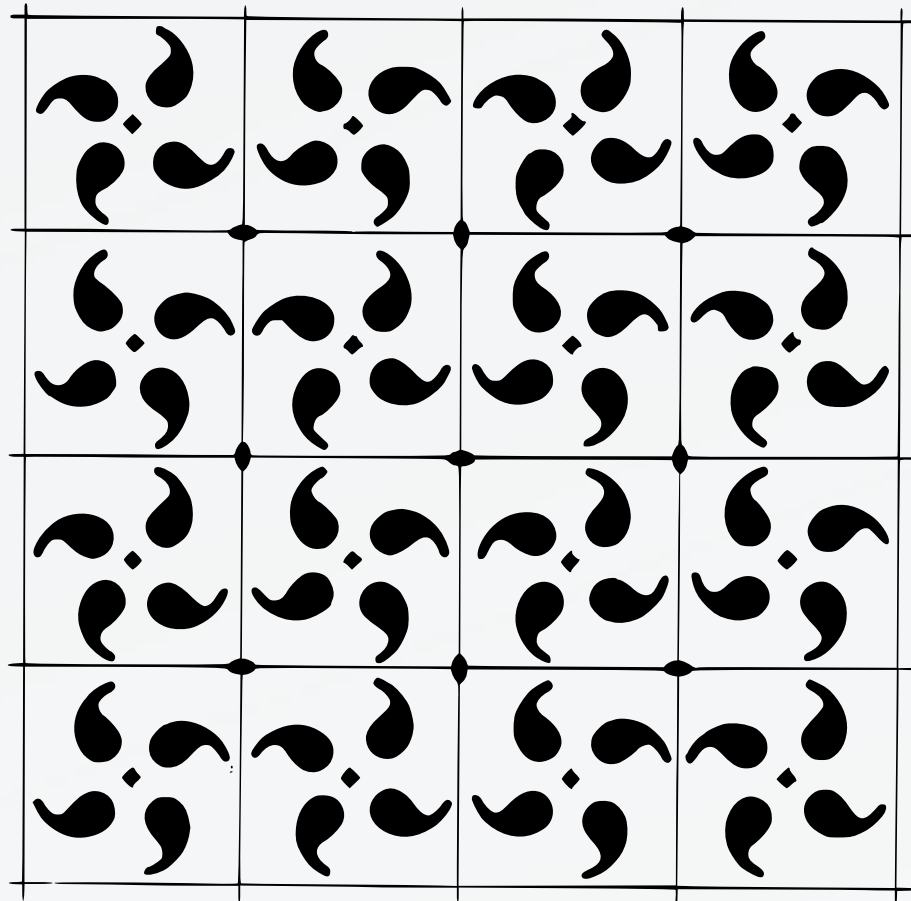


SQUARE

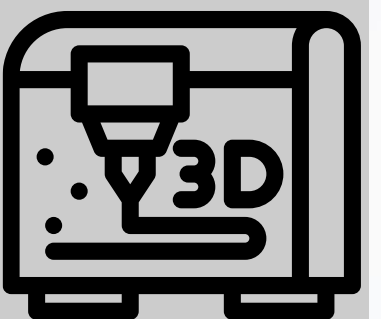
P4

P4MM

P4GM



P4GM



HEXAGONAL

P3

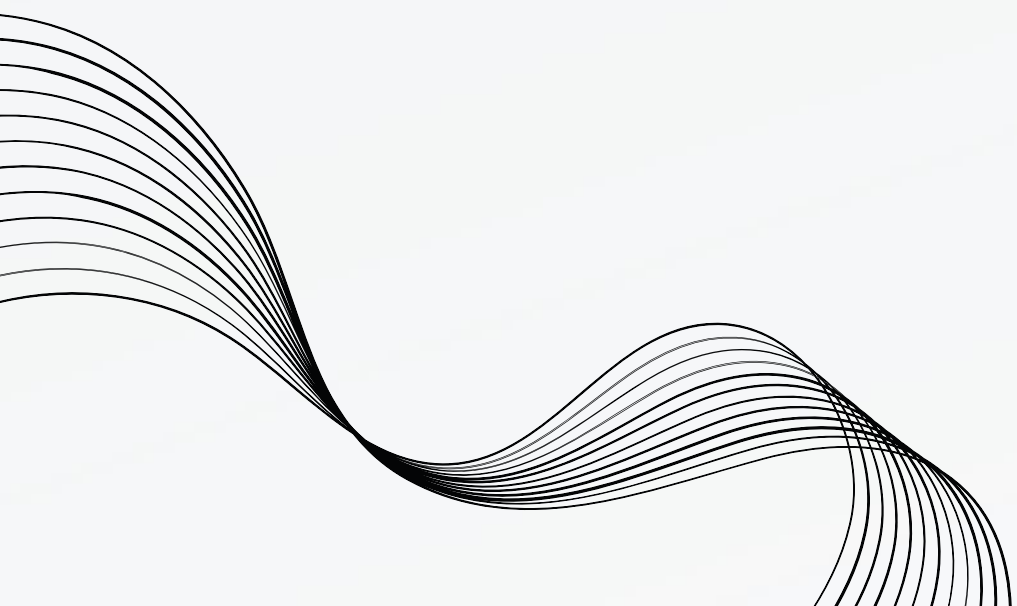
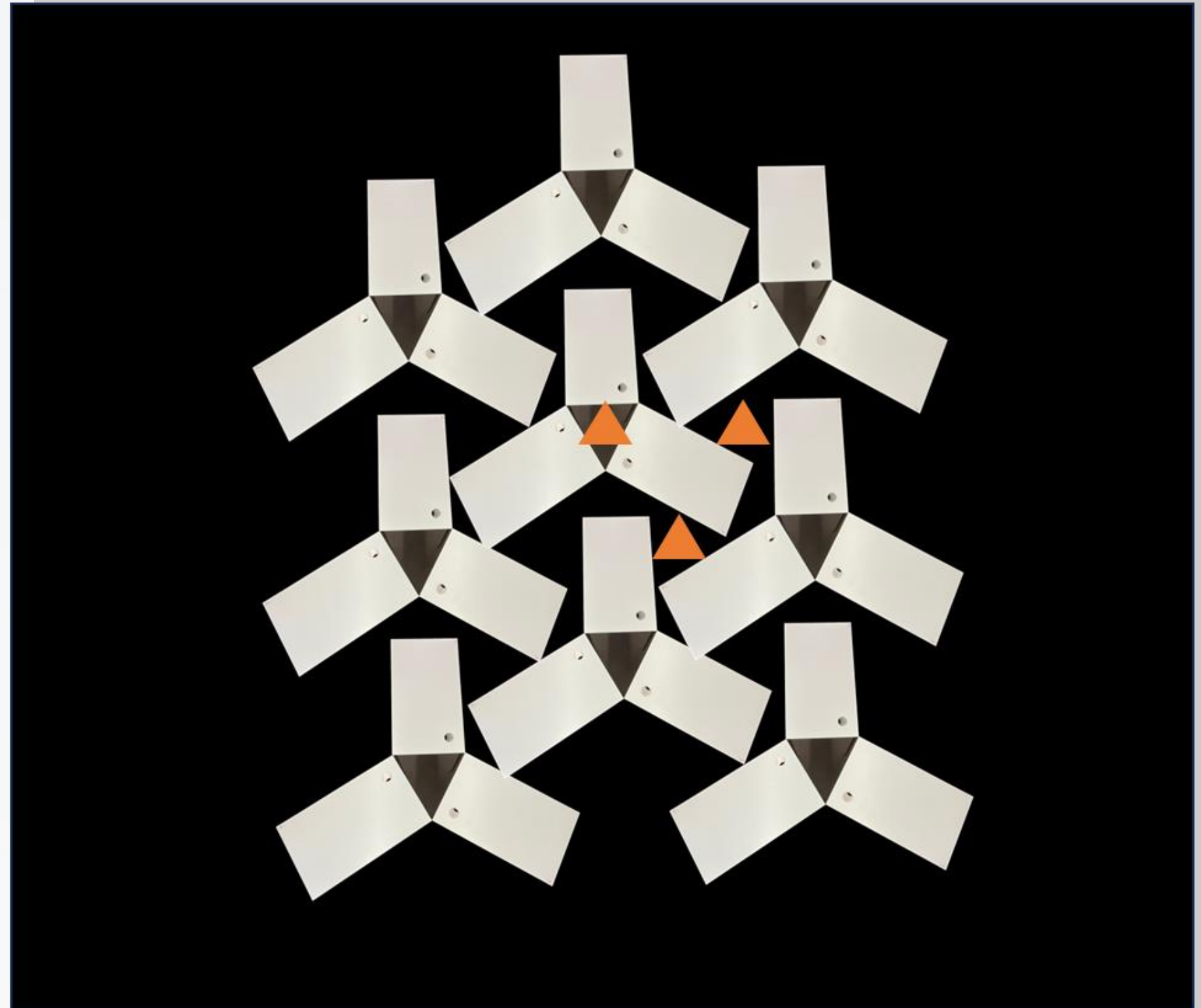
P3M1

P31M

P6

P6MM

P3



ANIMATION P3



HEXAGONAL

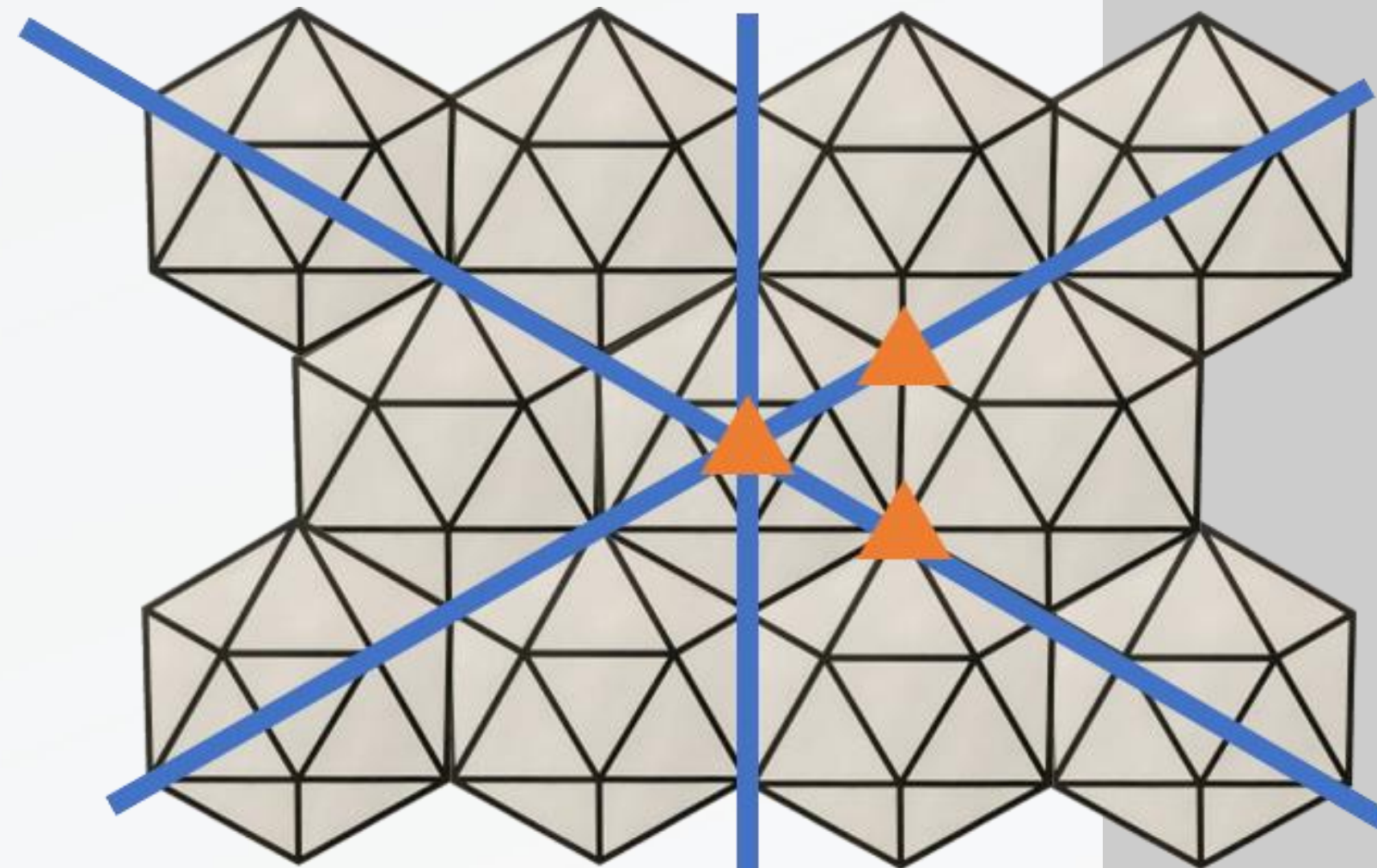
P3

P3M1

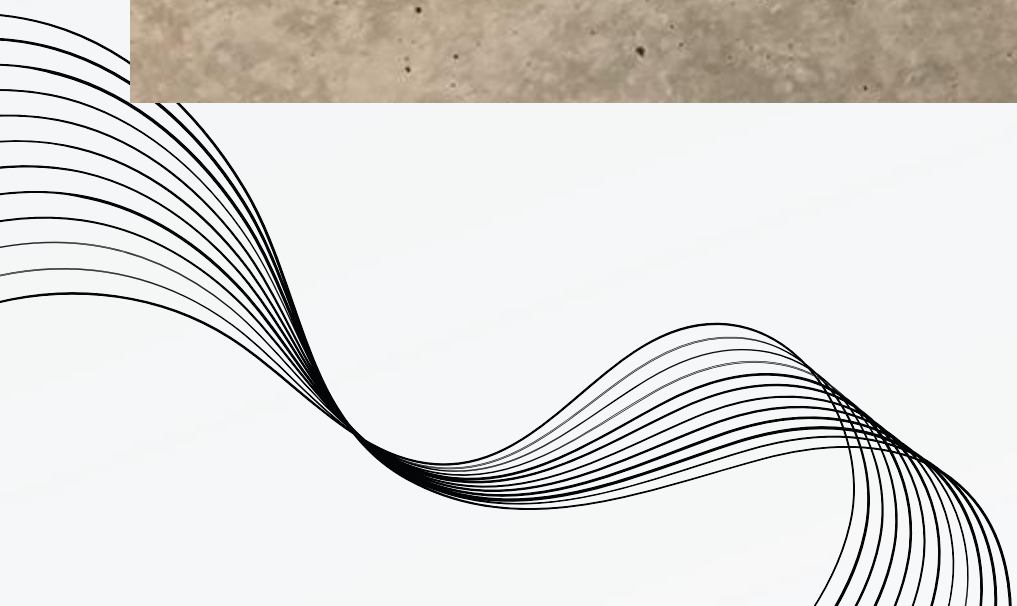
P31M

P6

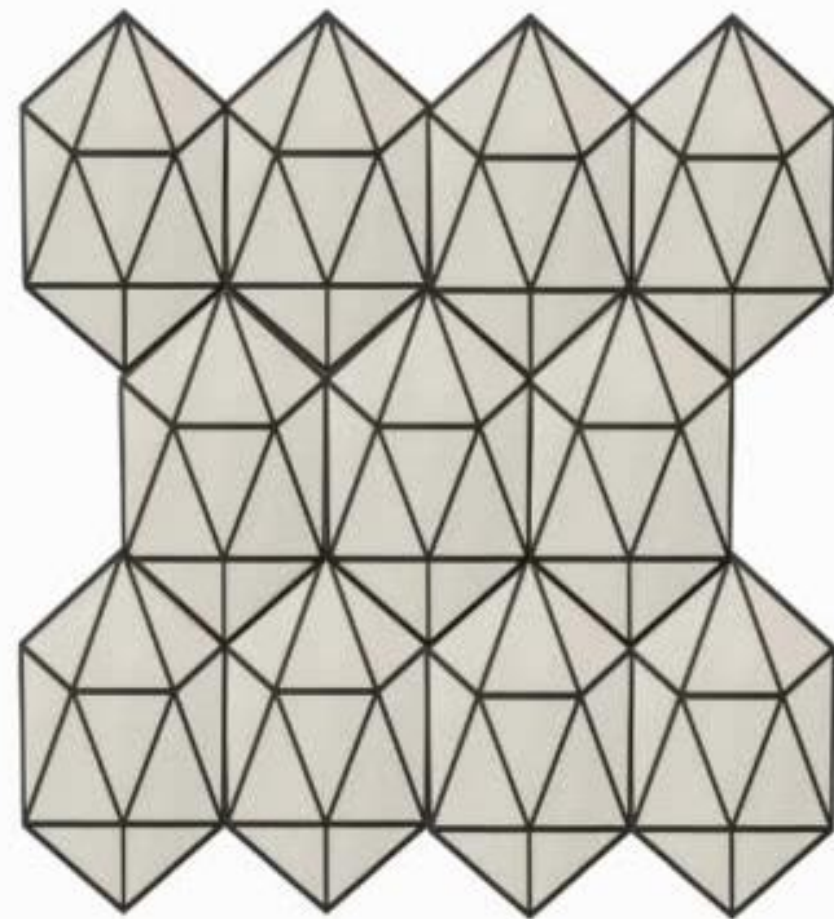
P6MM



P3M1



ANIMATION P3M1



HEXAGONAL

P3

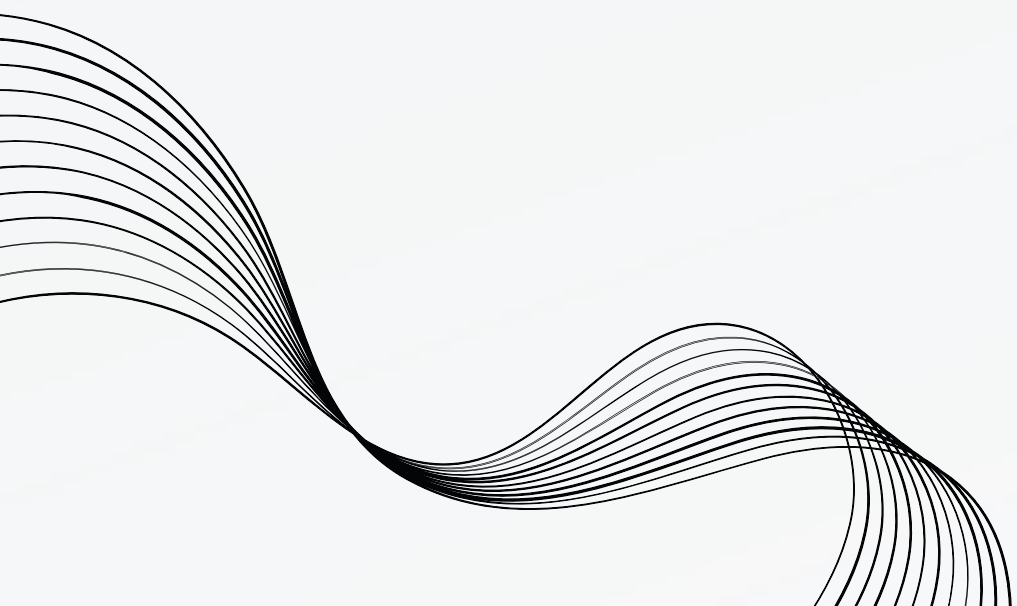
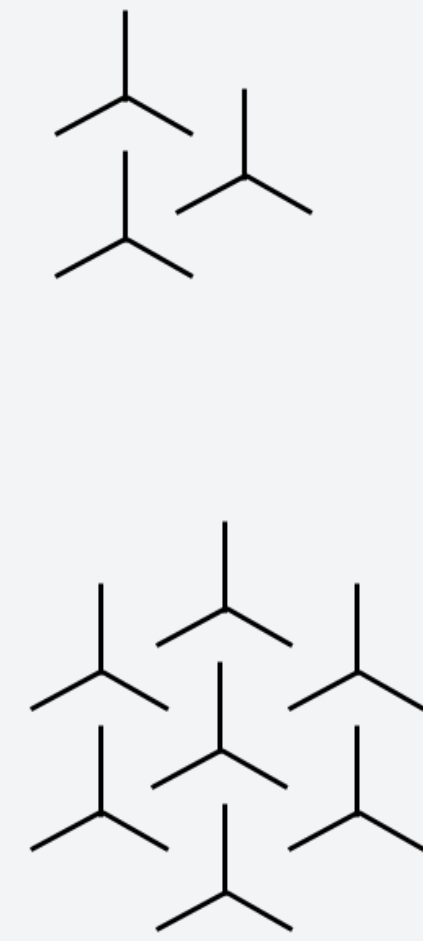
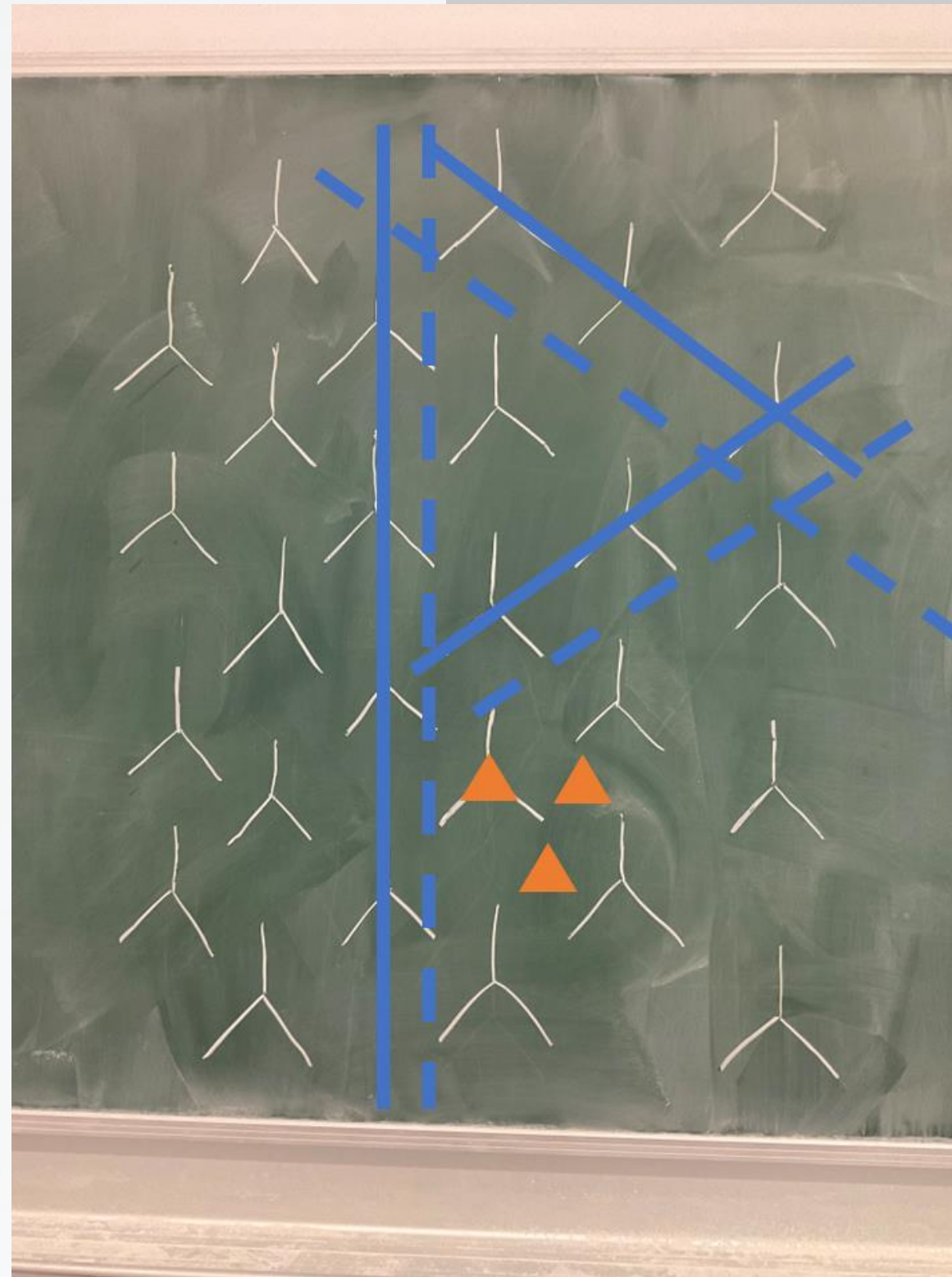
P3M1

P31M

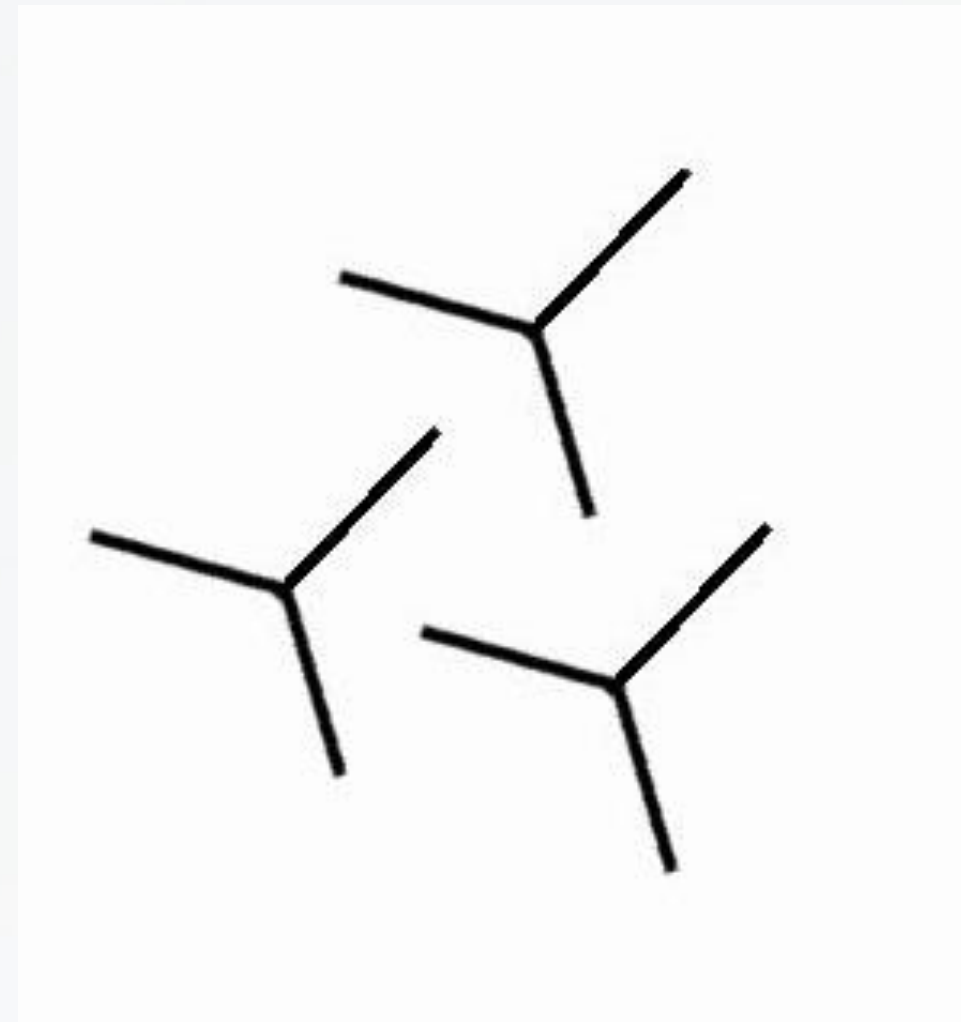
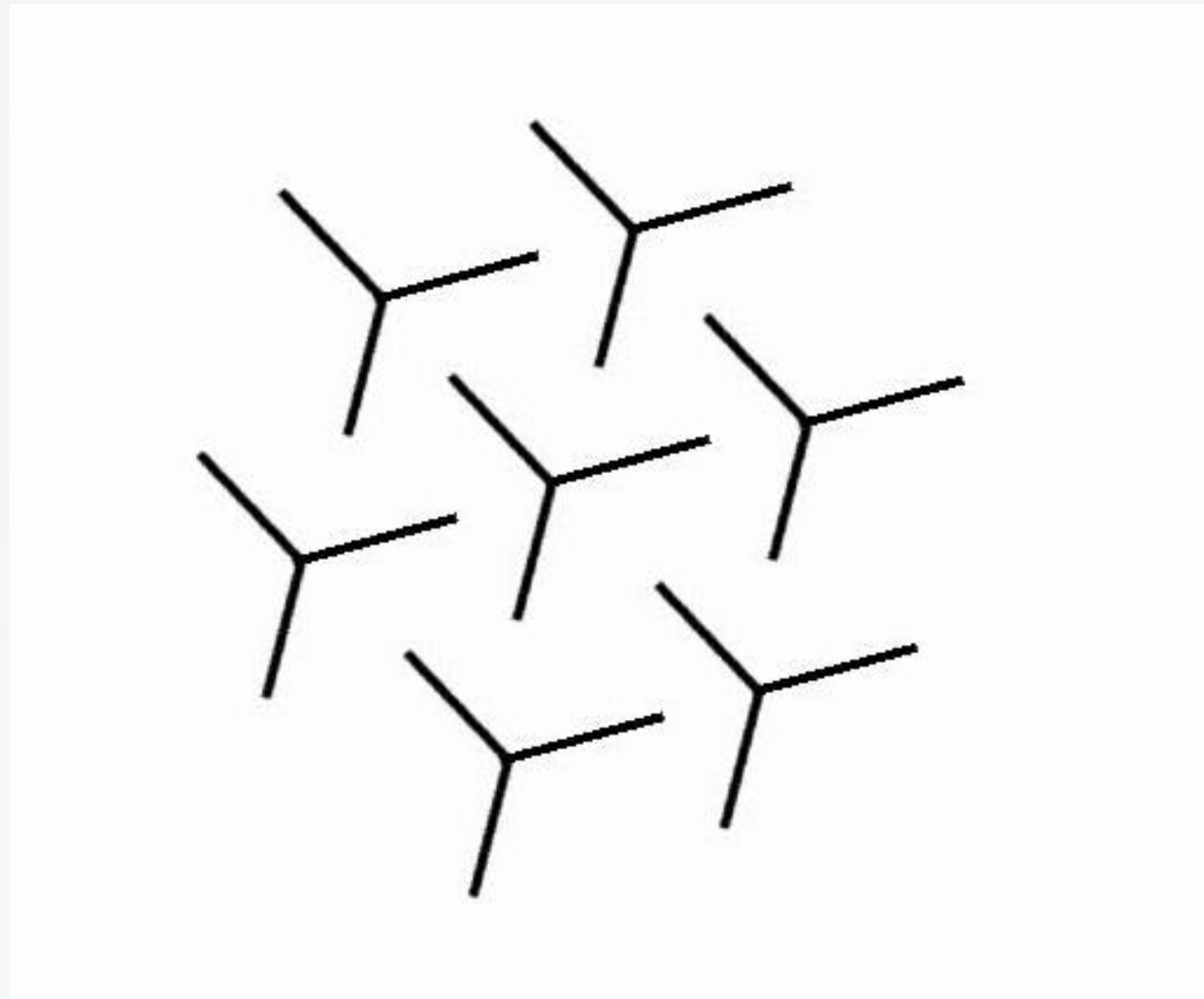
P6

P6MM

P31M



ANIMATION P31M



HEXAGONAL

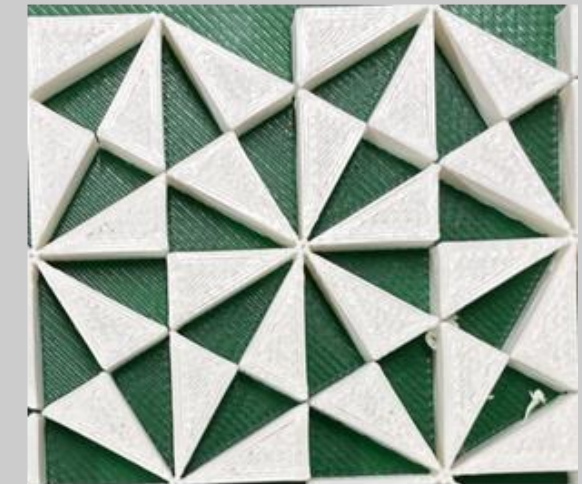
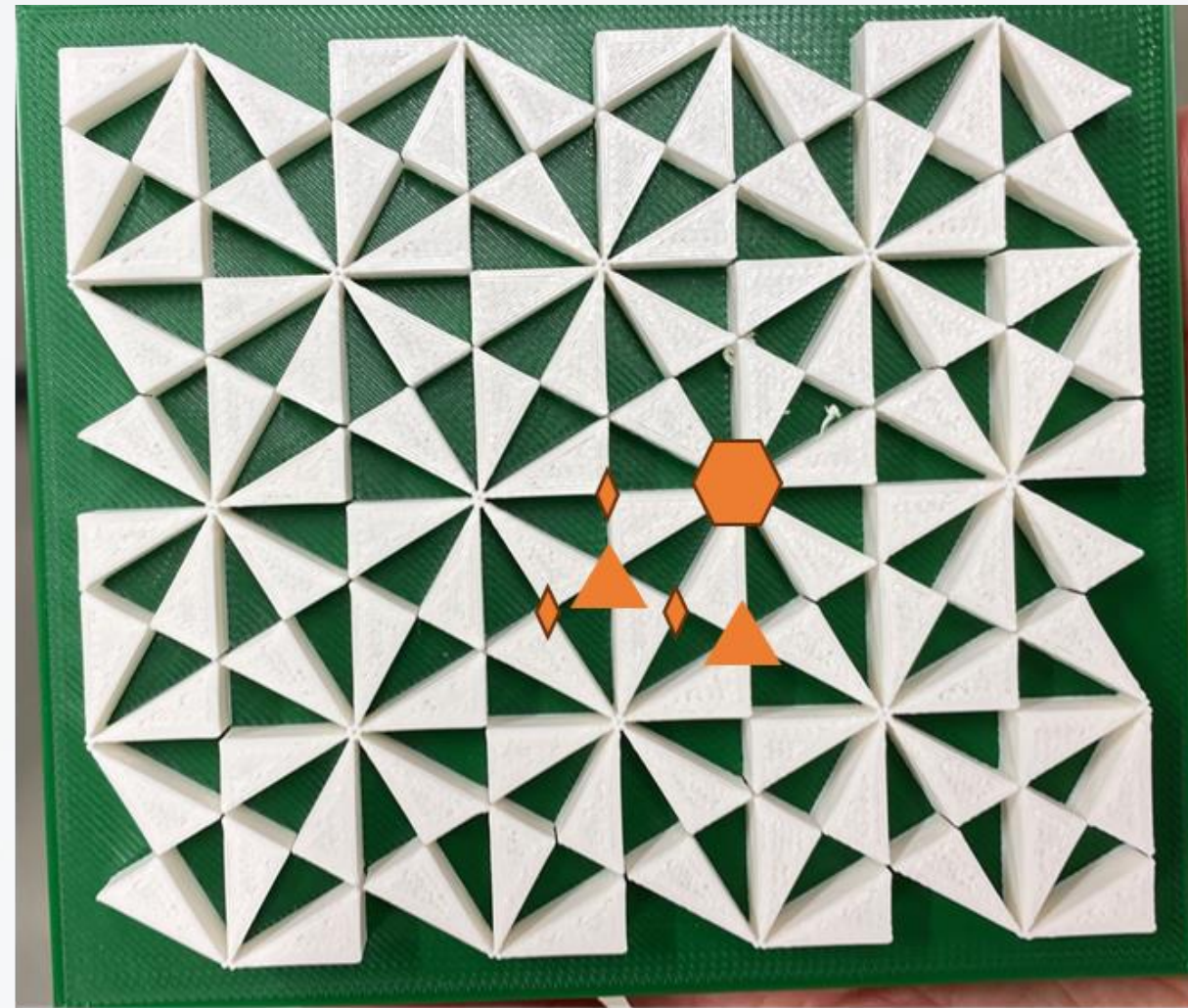
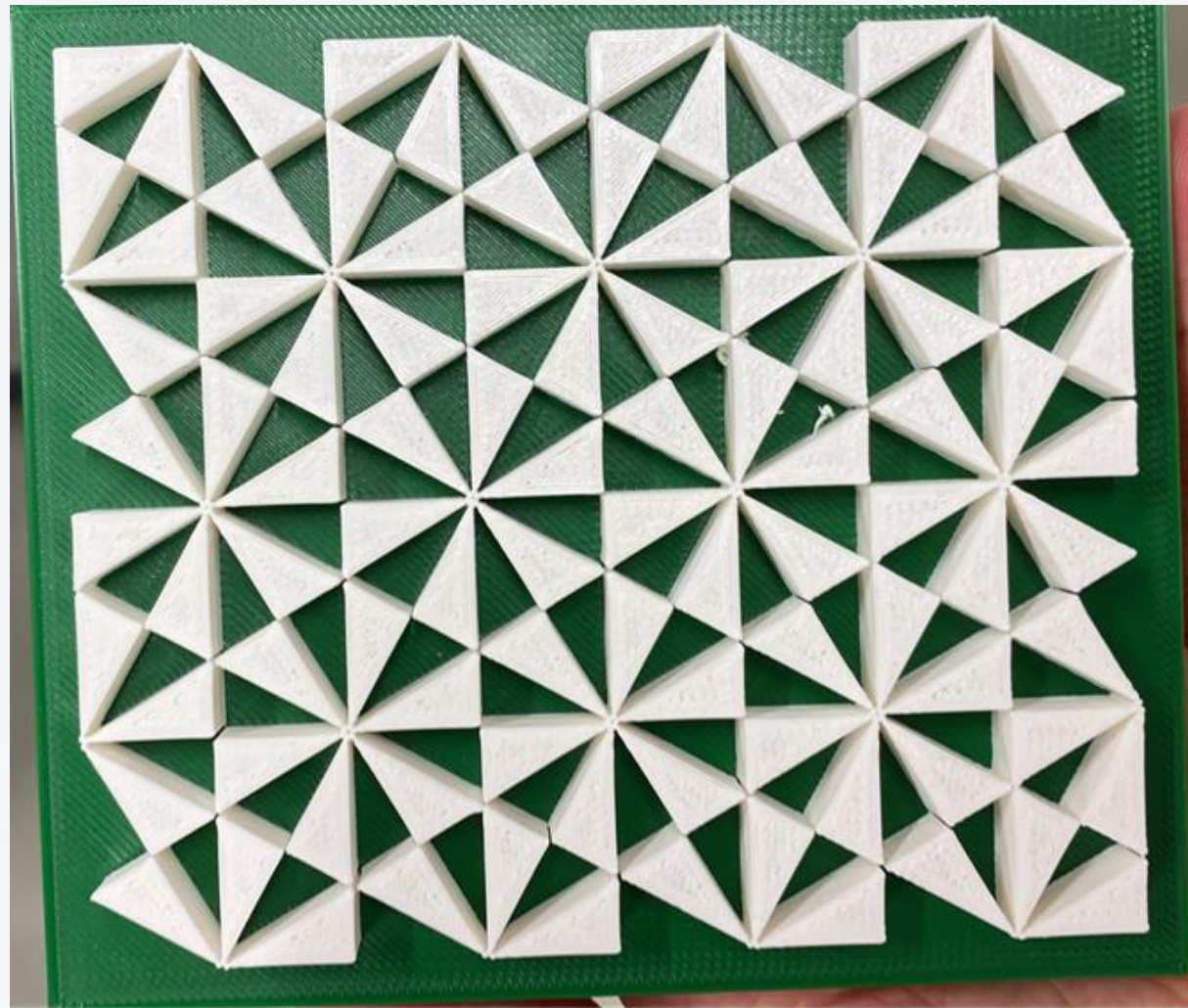
P3

P3M1

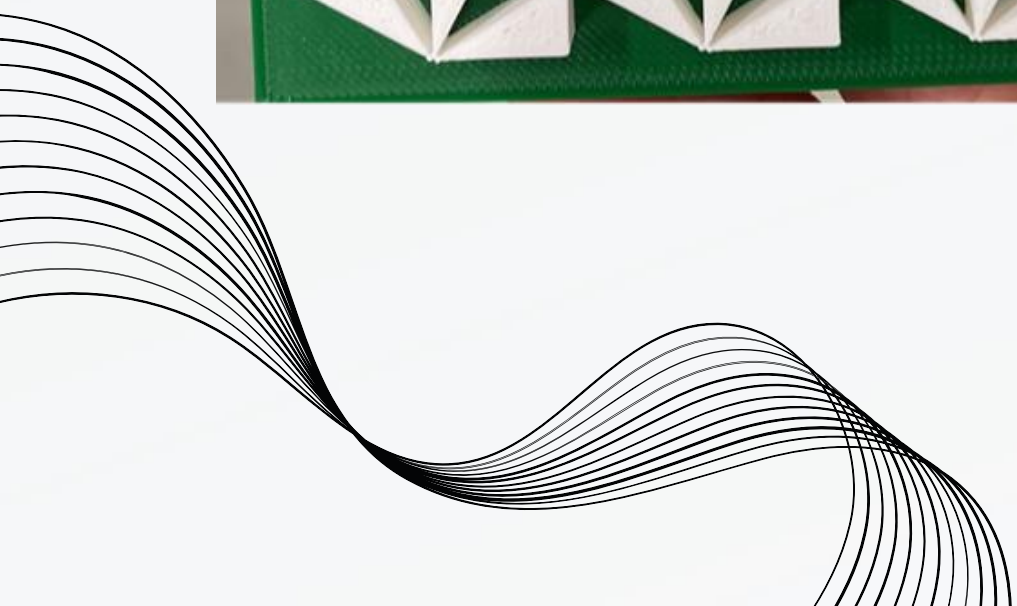
P31M

P6

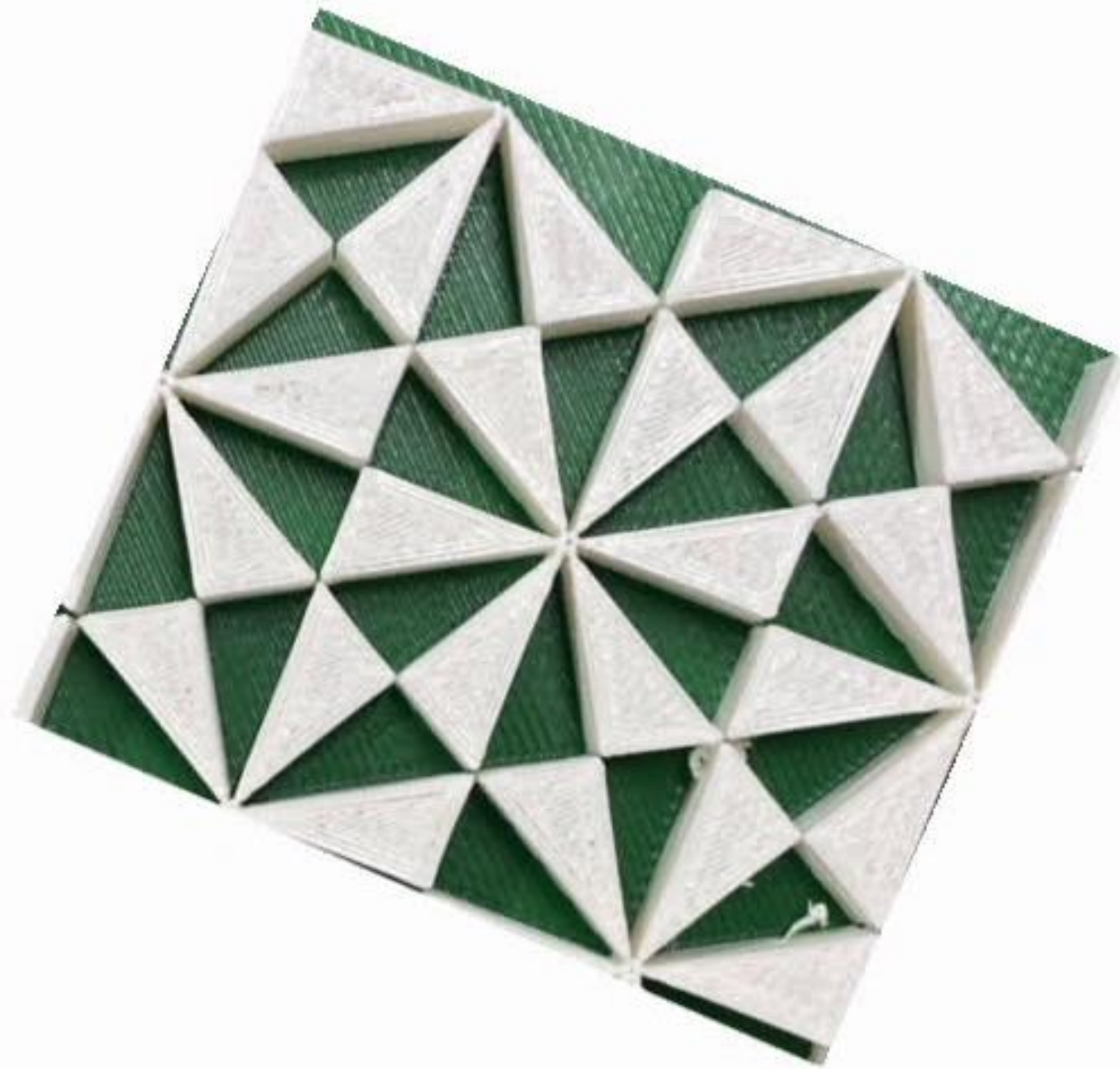
P6MM



P6



ANIMATION P6



HEXAGONAL

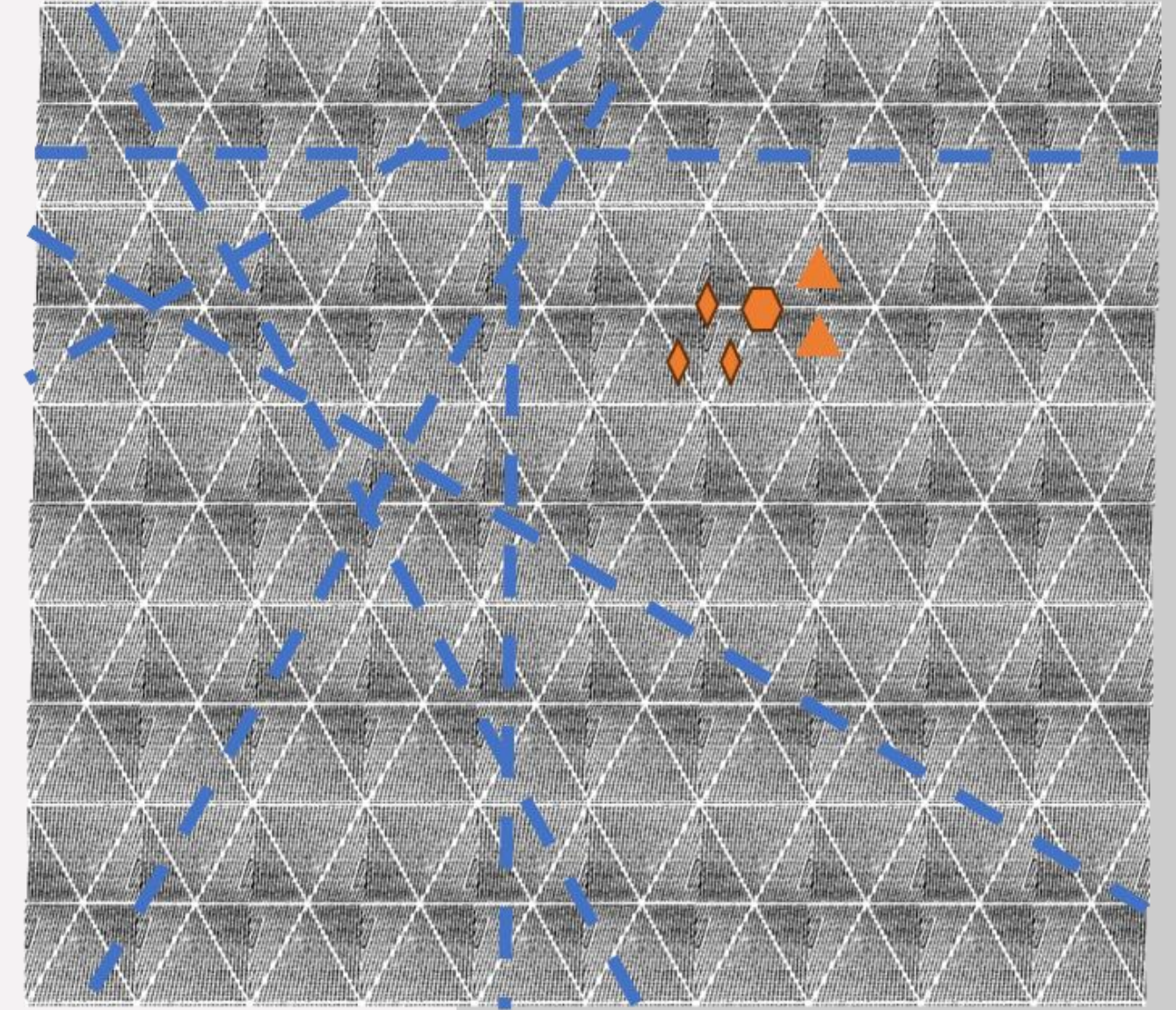
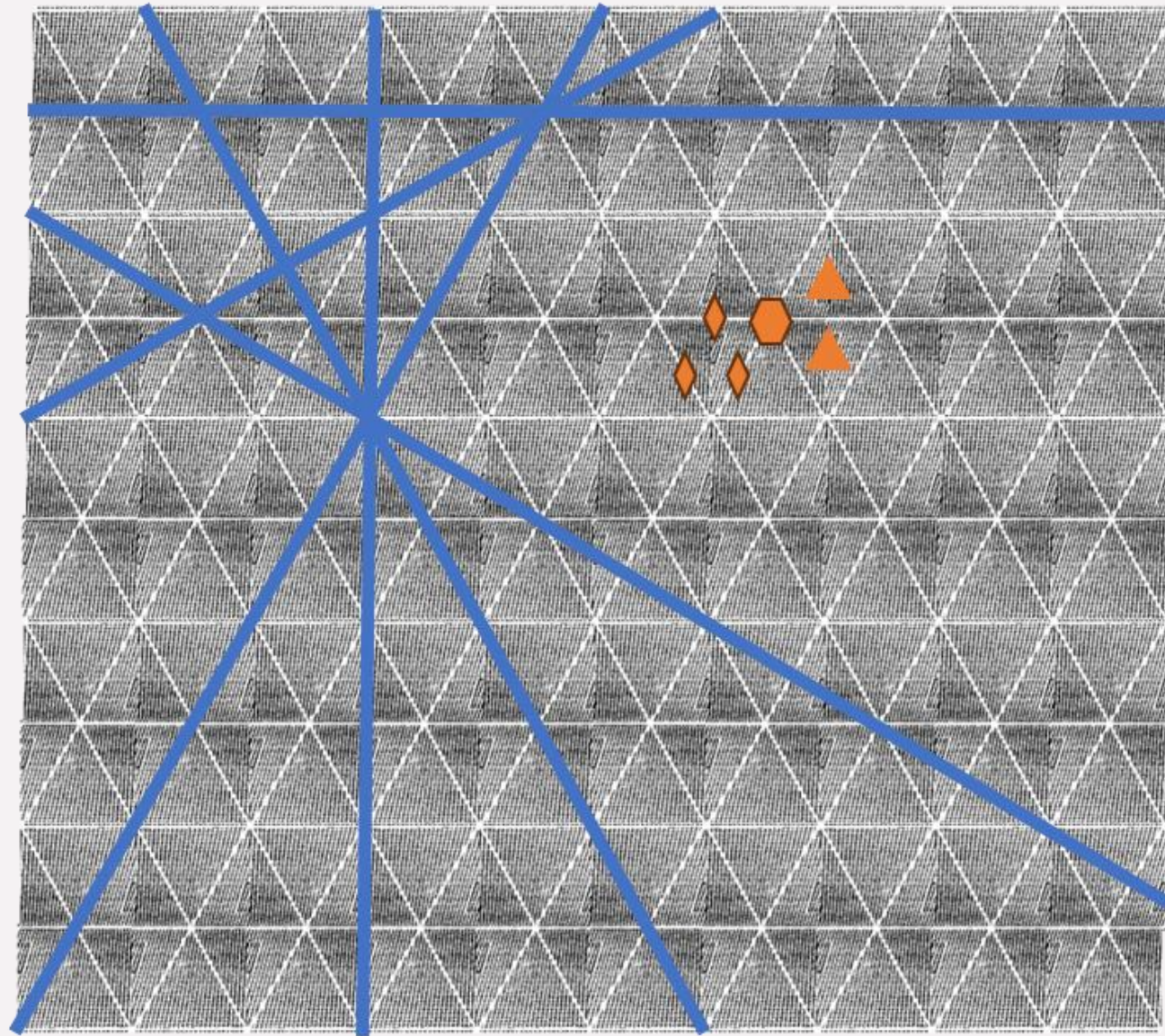
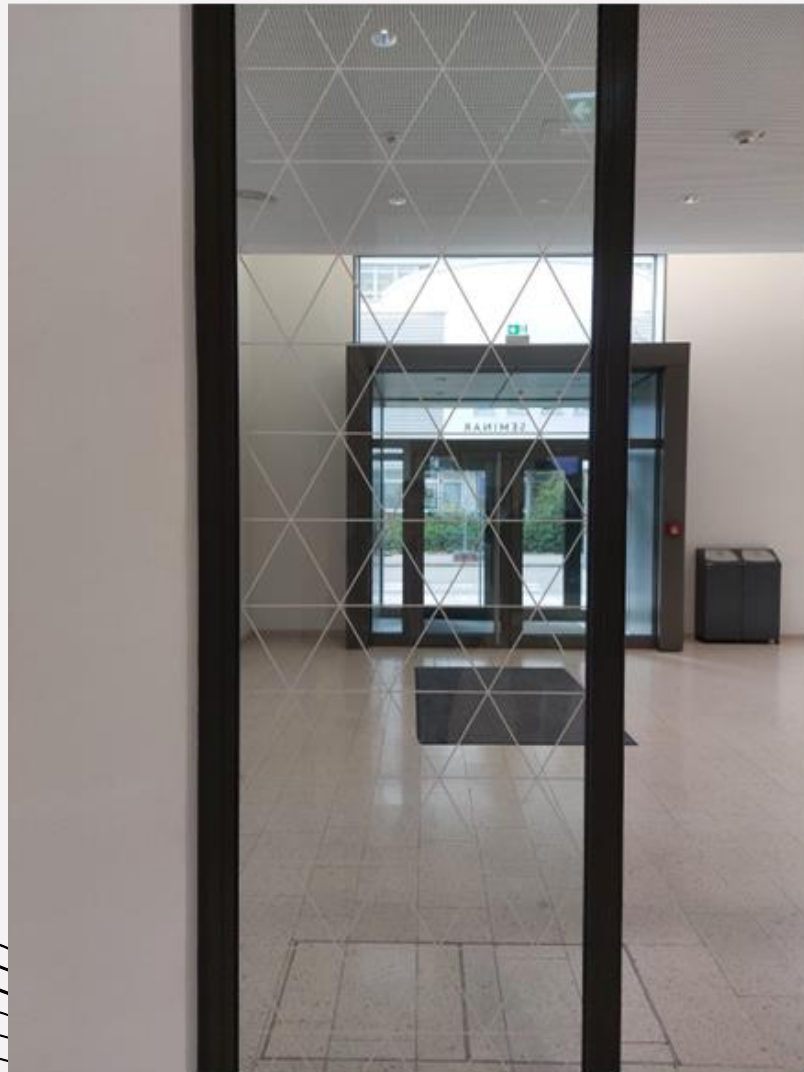
P3

P3M1

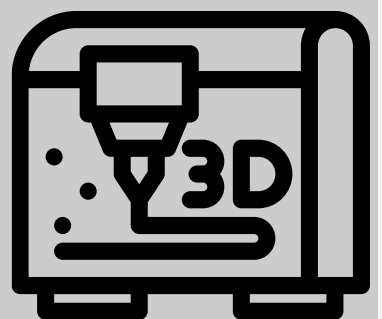
P31M

P6

P6MM

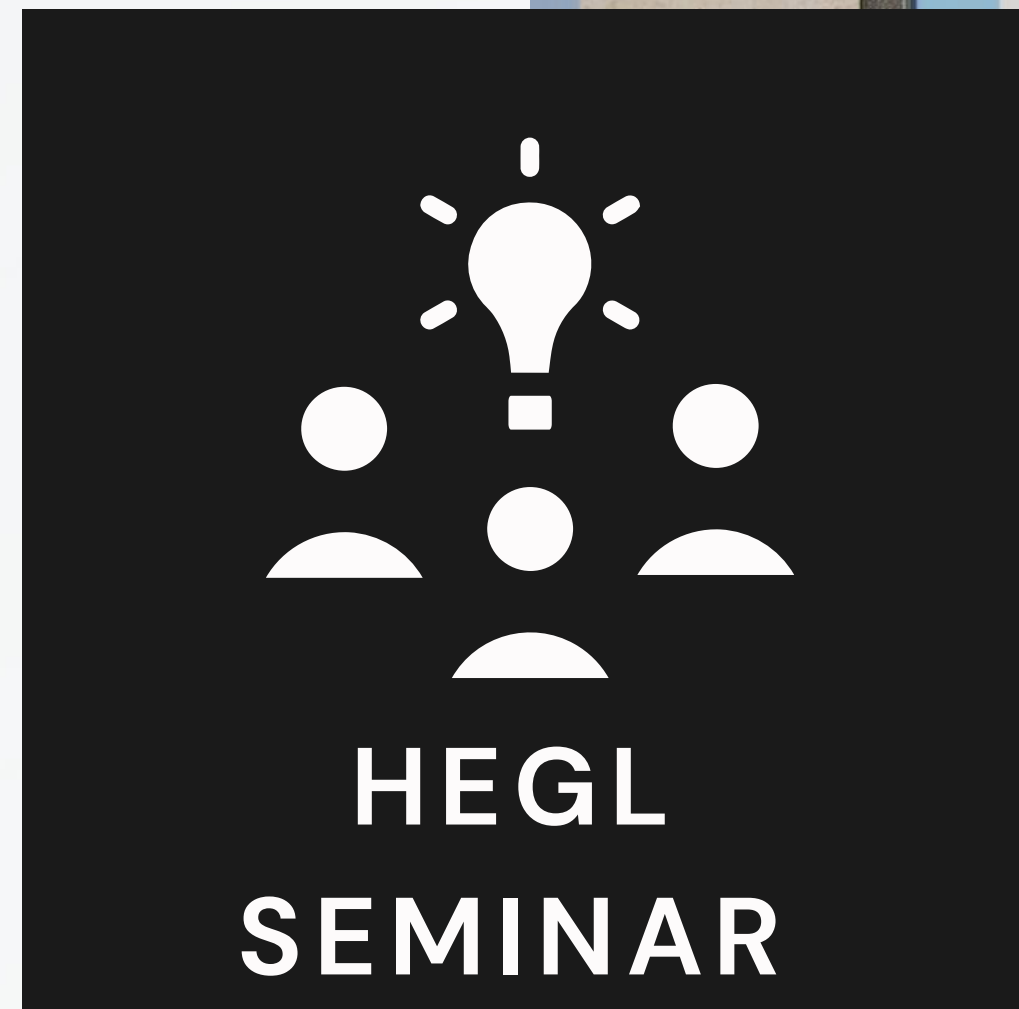


P6MM



YES, THE MATHEMATIKON CAN DO BETTER!

- there's no need to travel to Spain - just stay in Heidelberg & visit the Mathematikon



SOURCES

Photos:

- Wallpaper group examples
 - <https://mathworld.wolfram.com/WallpaperGroups.html>
- Alhambra:
 - https://structures.uni-heidelberg.de/blog/posts/2023_01_palace_crystallographic/images/fig13.jpg
 - https://upload.wikimedia.org/wikipedia/commons/d/de/Dawn_Charles_V_Palace_Alhambra_Granada_AndalusiaSpain.jpg
 - https://structures.uni-heidelberg.de/blog/posts/2023_01_palace_crystallographic/images/fig12.jpg
- Mathematikon:
 - https://www.katrinschacke.de/wp-content/uploads/2023/02/IM8A6649_logo-scaled.jpg

3D-files:

- <https://www.york.ac.uk/depts/maths/histstat/symmetry/wallpapers.htm>

P3M1 vs. P31M:

- <https://tessellations.ca/2015/06/09/p31m-vs-p3m1/>

Seminar literature:

- Armstrong, M. A., Groups and symmetry. Springer, New York (1988). <https://link.springer.com/book/10.1007/978-1-4757-4034-9>